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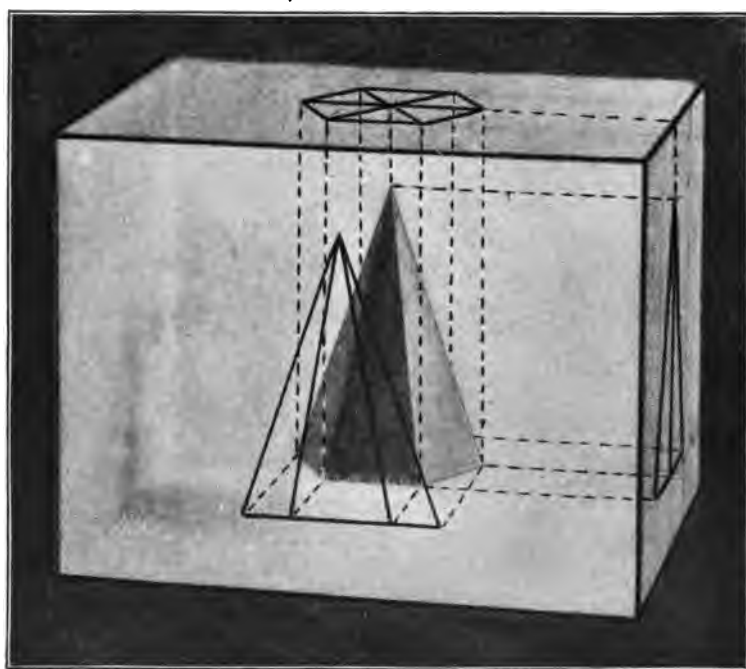


**FIG. 1.—THE VIEWS OF AN OBJECT ON THE TOP, FRONT, AND
SIDE PLANES OF PROJECTION.**

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DESCRIPTIVE GEOMETRY

FOR

THE USE OF STUDENTS IN ENGINEERING 36
IN HARVARD UNIVERSITY

BY

JAMES AMBROSE MOYER

INSTRUCTOR IN DESCRIPTIVE GEOMETRY IN HARVARD UNIVERSITY



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DESCRIPTIVE GEOMETRY

INTRODUCTION

1. *Descriptive Geometry* treats of the methods of making drawings to represent objects on two or more imaginary planes placed at right angles to each other. The drawings made by applying the methods of descriptive geometry show the object in the same way as in mechanical drawing. The object is represented as though the eye were infinitely distant, that is, the vanishing of the lines of the object in the distance is not represented.

The methods of *descriptive* or *practical geometry* are useful to engineers and architects in many structural and mechanical operations. In designing and constructing buildings and machines, it is often necessary to find the true size and shape of parts shown on drawings. When the necessary dimension lines are not shown, and when some of the actual lengths are fore-shortened, indirect geometrical methods of measurement must be used. The study of this subject is useful, however, for more than its industrial utility. The student becomes accustomed to considering very complicated geometrical combinations, and to follow with precision the correspondence between the drawings and the objects represented. This is really exercising in the most precise manner the power to visualize, which consists in representing to ourselves clearly and easily ideal objects as if they were really before us. "Descriptive Geometry," a well-known engineer has said, "trains one to see around corners."

CHAPTER I

ELEMENTARY PRINCIPLES AND NOTATION

2. A drawing of an object made by applying the methods we shall use is called an *orthographic projection*. We imagine the drawing is made on two or more transparent planes placed around the object as shown in Figs. 1 (frontispiece) and 2, where three views of a pyramid are represented on horizontal and vertical planes. These planes are called respectively the *top*, *front*, and *side planes of projection*. The top plane is horizontal and the front and side planes are vertical. These planes are supposed to be of indefinite extent. The top and front planes are the ones most used.

3. In order to represent the several views of the object on the plane of the drawing, the top and side planes are revolved about the line forming their intersection with the front plane till they are in the plane of the drawing.

In Figs. 1 and 2, the planes of projection are shown in position as we must imagine them placed in space. Fig. 2 also shows the lines XX and ZZ , which are the axes about which the top and side planes are revolved to make them coplanar, as shown in Fig. 3. The lines XX , YY , and ZZ are used as the axes of coördinates as in analytic geometry, and distances along these axes are represented respectively by the coördinates x , y , and z .

Distances along the XX axis (represented by the x coördinates) are measured in space, from the side plane and to the left or right of the YZ axis on the drawing.

Distances along the YY axis (represented by the y coördinates) are measured behind the front plane in space, and above the XX axis on the drawing.

Distances along the ZZ axis (represented by the z coördinates) are measured below the top plane in space, and below the XX axis on the drawing.

The solution of problems in orthographic projection requires of the imagination two distinct processes: (1) To conceive each view

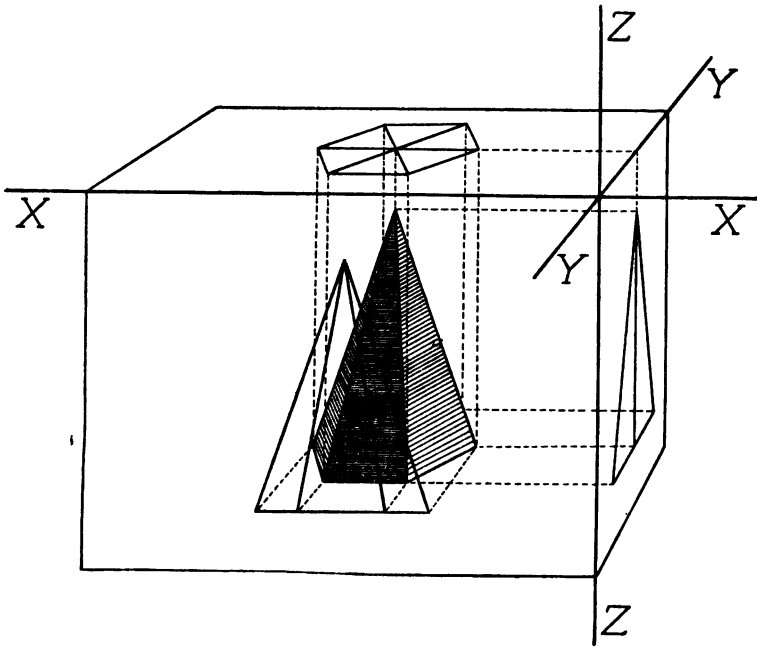


FIG. 2

in its corresponding plane. (2) To determine in the drawing the positions of the different views with reference to the XX , YY , and ZZ axes.

POINTS AND LINES

4. In orthographic projection the views shown by the drawing are the outlines of the object made by rays of light perpendicular to a plane of projection, reflected from points on the object. The points where the rays pierce the plane are called the *projections* of the corresponding points on the object. In Figs. 1–3 thus the lines of the outlines on the planes of projection are the projections of the edges of the pyramid. In the drawing in Fig. 3 any point on the pyramid such as a corner, a , of the base is shown by the projections a' , a'' , and a''' . These are respectively the *top*, *front*, and *side projections* of the point.

The top projection of a point is as far above the XX axis as the point itself is behind the front plane. The front projection of a point is as far below the XX axis as the point itself is below the top plane. A point lying in either the top or front planes has one of its projections coincident with the point itself and the other projection will lie in the XX axis. The top and front projections must lie in a straight line perpendicular to the XX axis, and the side and front projections in a straight line parallel to the XX axis.

5. A line is projected upon any plane by projecting all its points upon the plane. A straight line is determined in space by two points and the projections of these two points determine the projections of the line.

PLANES

6. Besides the top, front, and side planes, other planes must often be represented in the drawing. They are shown by their intersections with the planes of projection just mentioned. The lines showing these intersections of the plane are called its *traces*; and their slopes are represented usually by the angles they make with the XX axis. The angles are measured in anti-clockwise direction from 0° to 360° , as in trigonometry. Fig. 4 shows a plane which intersects the top and front planes of projection. The intersection of this plane with the top plane is its *top trace*, and its intersection with the front plane is its *front trace*. On all drawings these two traces must intersect on the XX axis, and testing for this intersection is a check on the accuracy of the constructions.

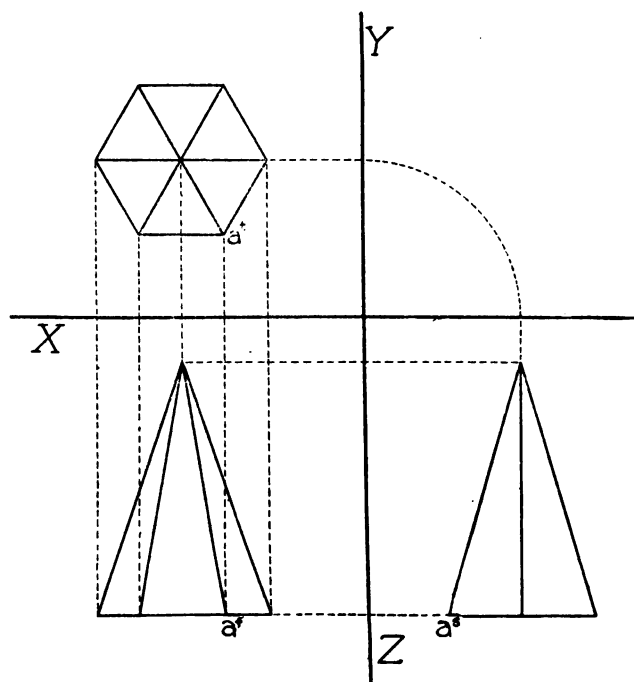


FIG. 3

NOTATION

7. A *point* in space is designated in the text by a small letter, and its projections by the same letter with t , f , or s placed above it, thus the top, front, and side projections of a point, a , are marked respectively a' , a'' , and a''' .

A *line* is denoted by two or more points in the line, thus a line containing the two points a and b is called the line ab .

The *traces* of a plane are designated by the same letter used in naming the plane with T , F , or S written before it, thus the top, front, and side traces of a plane, P , are marked respectively TP , FP , and SP .

8. Lines and traces of planes, given or required, are represented by heavy full lines when visible, by dotted lines when invisible. Projecting lines are indicated by short dashes. Traces of auxiliary planes are represented by long and short dashes.

9. Points are located by the coördinates x , y , and z (cf. Art. 3). A point a for which $x = 2$ (measured to the right of the YZ axis), $y = 3$ (the projection of the point in the top plane and measured above the XX axis), and $z = 4$ (the projection of the point in the front plane and measured below the XX axis) is represented thus: $a(2, 3, 4)$.

The point where a line intersects the top plane of projection is marked by the letters ti ; and the intersection of a line with the front plane by fi . To locate these points more plainly draw a small circle around their projections as shown in Figs. 5–8, on pages 13 and 17.

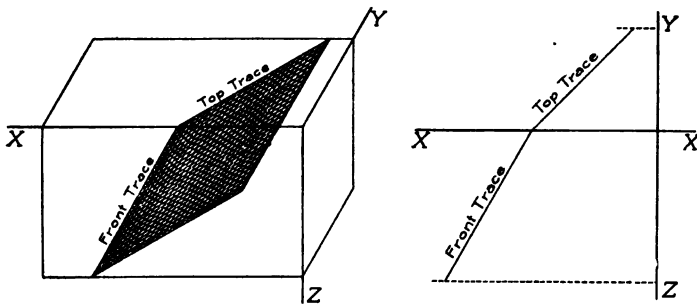


FIG. 4

CHAPTER II

PROBLEMS RELATING TO THE POINT, LINE, AND PLANE

10. The relation between the actual position of a line in space and its projections on a drawing is easily conceived for a line that is parallel to a plane of projection. Difficulty is, however, usually experienced in conceiving this relation for a line that is oblique to all the planes of projection. Fig. 5 shows a perspective drawing of such a line. The projections of a line on the planes of projection are drawn by joining the projections of any two points in the line. For the line shown in the figure, the points most easily determined in the drawing are its intersections with the top and front planes. These intersections are marked a' and b' . The intersection a' has its top projection coincident with the point itself, and its front projection, of course, in the XX axis at a' . Similarly the intersection b' has its top and front projections at b' (in the XX axis) and b' . The top and front projections of the line are then drawn by joining a' with b' and a' with b' .

11. By reversing the process the intersections of the line with the planes of projection can be found when the projections of the line are given, thus in Fig. 5 the intersection of the line ab with the top plane is the point a , shown by its top projection a' (over the intersection of the front projection of the line with the XX axis) and its front projection a' (on the XX axis). The intersection of the same line with the front plane is the point b , shown similarly by its projections b' and b' .

When only two planes of projection are mentioned (as is the case for most problems), it is assumed the side plane is not used.

12. **PROBLEM 1.** *To draw the projections of a line having given the intersections of the line with the top and front planes.*

Method. Draw the top projection of the line by joining the top projections of the two intersections. Draw the front projection of the line by joining the front projections of the two intersections.

Exercises

For notation see Art. 9. The coördinates of points are expressed in inches.

1. (a) A line intersects the top plane at the point $a (-3, 2, 0)$ and the front plane at the point $b (-1, 0, 2)$. Draw the projections of the line.
- (b) A line passes through the point $e (-2, 2, 1\frac{1}{2})$ and intersects the top and front planes at $o (0, 0, 0)$. Locate the projections of the line.

13. PROBLEM 2. *To find the intersections of a line with the top and front planes having given its projections.*

Method. Produce if necessary both projections to the XX axis. The intersection of the top projection of the line with the XX axis is one projection of the intersection of the line with the front plane. The other projection is where a line perpendicular to the XX axis through the projection found meets the front projection of the line. Similarly the intersection of the front projection of the line with the XX axis is one projection of the intersection of the line with the top plane. The other projection is where a line perpendicular to the XX axis through the projection found meets the top projection of the line.

Exercises

2. Find the intersections with the top and front planes of a line through the points $c (-3, 1, 2)$ and $d (-1, 2, \frac{1}{2})$.
3. (a) Draw the projections of two lines, one parallel to the top plane and oblique to the front plane, the other parallel to the front plane and oblique to the top plane.
- (b) Find the intersections of each line with the top and front planes of projection.
4. On the line given in Ex. 2, locate one point 1 inch below the top plane and another point $1\frac{1}{2}$ inches behind the front plane. Show both projections of each point located.
5. Given the line through the points $e (-3, 1, 2)$ and $f (-1, 2, 1)$. Locate its projection on the side plane.
6. Assume any line in space, and draw its projections. Find the new projections of this line (a) when the XX axis is perpendicular to its former position (coinciding with the YY axis of projection), (b) when the XX axis is revolved 30° (anti-clockwise) from its first position.

14. A line lying in a given plane intersects the top plane of projection at a point in the top trace, that is, in the line showing the intersection of the given plane with this plane of projection. The same line intersects also the front plane in the front trace of the given plane. Obviously then the top projection of the intersection of a line with the top plane is in the top trace of the given plane; and the front projection is in the XX axis. Similarly the top and front projections of the intersection of the line with the front plane are respectively in the XX axis and in the front trace of the given plane. This is illustrated in Figs. 7 and 8, where a line lying in a plane is shown. The line is produced at both ends to indicate its intersections with the top and front planes. The two projections of the line are also drawn in Fig. 7. In Fig. 8 only the front projection, $a'b'$, of the line is given. The other projection of the line is found by determining the points of intersection of the line with the top and front planes, ti and fi ; and then joining the top projections of these points, fi' and ti' .

15. PROBLEM 3. *Given one projection of a line, or a point, in a given plane, to find the other projection.*

Method. (a) Given one projection of a line. Determine the positions of the intersections of the line in the given plane with the top and front planes, and draw the required projection.

Method. (b) Given one projection of a point. Draw a projection of any line through the given projection of the point. Find the other projection of this line as described above. On this projection of the line, vertically over the given projection of the point, is the required projection of the point.

Exercises

7. The line through a ($-2, 1, ?$) and b ($-1, \frac{1}{2}, ?$) is in a plane intersecting the XX axis at $x = 0$, with its top and front traces making angles respectively of 150° and 240° with the same axis. Draw the front projection of the line.
8. Given one projection of a line oblique to the XX axis in a plane with traces parallel to this axis. Find the other projection of the line.
9. Locate a point in the plane given in Ex. 7 that shall be $\frac{1}{2}$ inch behind the front plane. In the same plane, locate also a point $1\frac{1}{2}$ inches below the top plane.

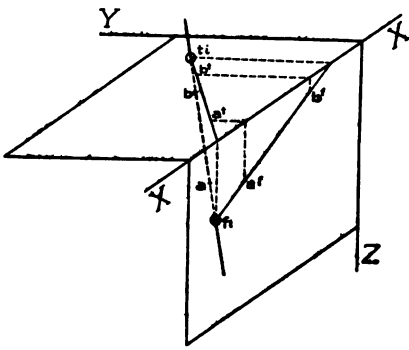


FIG. 7.

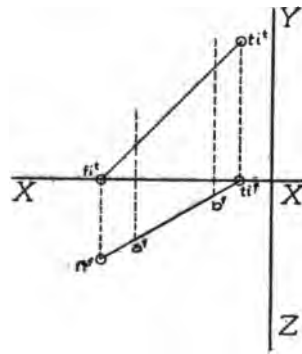


FIG. 8.

10. Draw the projections of a point in a plane which is perpendicular to the planes of projection.
11. Draw the projections of a point in a plane which is parallel to the top plane.
12. The slope of a roof on a building is represented by its top and front traces, making angles respectively of 150° and 225° with the XX axis. There is a hole in the roof for a chimney of which the top projection is shown in the drawings as a regular hexagon (length of sides is $\frac{1}{4}$ inch). The top projection of the centre of the hole is located $1\frac{3}{4}$ inches to the left of the intersection of the traces and $\frac{1}{2}$ inch above the XX axis. Draw the front projection of the hole.
13. A circular water pipe passes through an inclined floor represented on a drawing by top and front traces making angles respectively of 150° and 210° with the XX axis. The centre line of the pipe is parallel to the top plane. The front projection of the pipe is therefore a circle, and its centre is shown $1\frac{1}{2}$ inches to the left of the intersection of the traces and $\frac{1}{2}$ inch below the XX axis. Complete the drawing by finding the top projection of the hole through which the pipe passes. (Draw a number of diameters of the circle, producing them when necessary, and find the top projections of these lines. The points in the outline of the top projection of the hole will be vertically over the corresponding points in the given front projection.)

16. Lines that are parallel to a plane appear in their true length in their projections on that plane. In Fig. 9 (Plate I)* the top and front projections of a hip roof are shown. The lines that are parallel or perpendicular to the XX axis are here drawn in their true length. A line that is oblique, however, to both planes of projection, as for example the line bf , is not shown in its true length in the drawing; but a third view is made to show its true dimensions. This is illustrated in the right-hand drawing in the figure. The roof is now projected on a vertical plane, VV , drawn through the line bf , and then revolved into the plane of the drawing. This projection

* Figs. 9-33 are on the plates at the back of the book.

was constructed by drawing the base line, $V'V'$, parallel to VV . At g' on $b'b'$ produced the line $g'b'$ is laid off perpendicular to $V'V'$. Its length is equal to the "altitude" of the point b , or to the distance $g'b'$ in the front view. The line $b'f'$ shows then the true length of bf . The completed drawing shows also the true length of the line ac , as it is parallel to the plane VV .

17. In practice these constructions are simplified by passing the auxiliary vertical plane *through* $b'f'$. The line VV is the trace of this plane in the plane of the base of the roof. If this plane is revolved, together with the line bf , about VV to coincide with the plane of the base, the line bf will then lie in a plane where true dimensions can be measured.

The revolution of one line about another line as an axis is accomplished usually by the revolution of two points in the line. A point revolved thus about a line as an axis, describes a circle with a radius equal to the actual perpendicular distance from the point to the axis. In this case the points b and f must be revolved about the axis VV . The axis here is in the plane of the base so that the perpendicular distances of the points from it can be measured in the front view. For the point b this distance is $g'b'$ which is laid off on the line through b' , perpendicular to VV . The revolved position of b is at b'' . The point f is in the axis, so that in revolution it is stationary. The line $b''f'$ is then the position of bf when revolved into a plane of the drawing, and shows its true length.

18. If it is required to *lay off a given distance* on a line not shown in its true length in a drawing, we must first revolve the line about an axis to bring it into a plane where its true length is shown, and then measure the distance. For example, if we wish to measure a distance, bx along the line bf (Fig. 9) from the point b , we lay off the given length on the revolved position $b''f'$, locating the point x'' , and then reversing the preceding process, we revolve x back into the line bf . The top projection of bx is then $b'x'$.

19. Usually when the true lengths of lines are to be found it is most convenient to revolve them into either the top or front planes, rather than into an additional plane parallel to one of the planes of projection. A case sometimes occurs where the top and front planes have not been advantageously located, so that lines of the object

pass through and continue beyond these planes. Here then the projections of the parts of the line fall on opposite sides of the XX axis. When such a line is revolved into the plane which it passes through, distances must be laid off on opposite sides of the axis of revolution.

In Fig. 10 a line ac is shown intersecting and passing through the top plane at b . If we revolve the line into the top plane about $a'c'$ as an axis, the point b remains stationary, and a and c revolve in *opposite directions*. The distance that a' is below the XX axis is measured on one side of $a'c'$, and the distance that c' is above is measured on the other side. Then $a'c'$ is the true length of the line. If the same line is revolved into the front plane, this difficulty is not met, as $a''c''$ now shows the true length.

20. PROBLEM 4. *To find the true length of a line given by its projections. (To find the distance between two points.)*

Method. Pass a plane through the line perpendicular to a plane showing true lengths on the drawing. With the intersection of the two planes as an axis, revolve the line into the second plane, where it is shown in its true length.

Exercises

14. Find the distance between the points $a(-5, 1, \frac{1}{2})$ and $b(-1, 3, \frac{1}{2})$.
15. Find the true distance between the points $c(-6, 1, 2)$ and $d(-\frac{1}{2}, 2, \frac{1}{2})$.
16. Locate any two points, two inches apart, in a plane parallel to the front plane.
17. A line intersects the top and front planes at the points $e(-5, 0, 1\frac{1}{2})$ and $f(-1, 2\frac{1}{2}, 0)$ respectively. Find the length of the line included between the two planes of projection.
18. A line is $4\frac{3}{4}$ inches long. The projections of its ends are located by the points $a(-5, 1, \frac{1}{2})$ and $b(-1, 3, ?)$. Determine the projection that is not given.
19. Assume the data of Ex. 12 and through the center of the chimney draw a line, with its top and front projections perpendicular to the top and front traces of the roof plane. Lay off on this line $1\frac{1}{2}$ inches above the roof and $\frac{3}{4}$ inch below.

20. A line lies in the plane, M , intersecting the XX axis at $x = -3$, with its top and front traces making angles respectively of 30° and 330° with the axis. One projection of the line is given by the points $g(-1\frac{1}{2}, \frac{1}{4}, ?)$ and $h(-\frac{1}{2}, 1, ?)$. On this line lay off $1\frac{1}{2}$ inches from its intersection with the front plane.

21. A line lying in a given plane intersects the top and front planes of projection at points in the line marking the intersection of the given plane with the top and front planes. This is illustrated in Fig. 11 where the top and front projections of a pyramid are shown. We shall show the plane of the side abc by its traces. The lines ab and ac are in the plane of this side, and their intersections with the top and front planes respectively are marked ti and fi . These points then lie both in the given plane and in one of the planes of projection. In other words, they lie in the traces of the plane. The top trace, TP , of the plane is obtained by joining the two top intersections (ti); and the front trace, FP , by joining the two front intersections (fi).

In the example explained, the lines determining the plane intersected each other; but the solution is the same for parallel lines. A plane is determined also by any three points not in the same straight line, or by a point and a line. The first case, that of the three points, is most easily solved by joining them by straight lines "two and two." The second case, that of the point and line, by drawing through the point any line intersecting the given line. Both cases are then resolved into that of drawing a plane through two intersecting lines.

22. The problems of *drawing a plane through one line parallel to another, or through a given point parallel to two given lines* are scarcely more than variations of this same problem. In the first case, one line in the required plane is given and the direction of another. Through any point in the first line a line is drawn parallel to the second. In the last case, the directions of two lines are given, and the plane is determined by drawing two lines parallel to them through the given point. Both cases are resolved again into passing a plane through two intersecting lines.

23. PROBLEM 5. *To pass a plane through two intersecting or parallel lines.*

Method. Find the points of intersection of the given lines with the top and front planes. The line joining the intersections in the top plane is the top trace of the plane. The line joining the intersections in the front plane is the front trace.

Exercises

In the following exercises mark the top and front traces of a plane not otherwise designated by the letters *TP* and *FP*.

21. The line through $a(-2, \frac{1}{2}, \frac{3}{4})$ and $b(-1, 1\frac{1}{2}, \frac{1}{4})$ intersects the line through $c(-2, 1\frac{1}{4}, 0)$ and $d(-\frac{1}{2}, \frac{1}{2}, 1\frac{1}{2})$ at $e(-1\frac{1}{2}, 1, \frac{1}{2})$. Pass a plane through these two lines.
22. Draw the projections of any two lines *intersecting* at the point $m(-1\frac{1}{2}, 1, \frac{1}{2})$. Determine the plane in which these two lines lie.
23. Draw the traces of a plane through the *parallel* lines $a(-3\frac{1}{2}, \frac{1}{4}, 1)$, $b(-2\frac{3}{4}, \frac{3}{4}, \frac{1}{4})$; and $c(-1\frac{1}{4}, 1\frac{1}{4}, 1)$, $d(-\frac{1}{2}, 1\frac{3}{4}, \frac{1}{4})$.
24. Pass a plane through the three points: $a(-2, \frac{1}{4}, 1)$, $b(-1, 1, \frac{1}{4})$, and $c(-\frac{1}{2}, \frac{1}{4}, \frac{1}{4})$.
25. Find the plane passing through the line $d(-1, \frac{1}{4}, \frac{3}{4})$, $e(-\frac{1}{2}, \frac{3}{4}, \frac{1}{2})$ and the point $f(-2\frac{3}{4}, 1, -1)$.
26. (a) Through the line $g(-1\frac{1}{4}, \frac{1}{2}, 1)$, $h(-\frac{1}{2}, 1, \frac{1}{4})$ pass a plane parallel to the line $i(-2\frac{1}{2}, 1\frac{1}{4}, \frac{1}{4})$, $j(-1\frac{3}{4}, \frac{1}{2}, \frac{3}{4})$.
(b) Through ij pass a plane parallel to gh . (Observe that the corresponding traces of the two planes are parallel.)
27. Through the point $o(-1, 1\frac{1}{2}, \frac{1}{2})$ pass a plane parallel to the lines $k(-3\frac{1}{4}, 1\frac{1}{4}, 1)$, $l(-2\frac{1}{2}, \frac{1}{4}, 1\frac{1}{2})$; and $m(-2\frac{1}{2}, 1, \frac{1}{2})$, $n(-2, 2, 1)$.
28. Pass a plane which is parallel to the *XX* axis through the point $p(-2, 1, 1)$. Draw the trace of the plane on the side plane of projection.
29. The end posts of the portal of a skew bridge are shown on the engineer's drawings by the lines $r(-2, \frac{1}{4}, \frac{3}{4})$, $s(-1\frac{1}{2}, \frac{3}{4}, \frac{1}{8})$; and $t(-1\frac{1}{2}, \frac{1}{4}, \frac{3}{4})$, $u(-1, \frac{3}{4}, \frac{1}{8})$. The top projection shows the plan and the front projection the elevation. Draw on the three planes of projection, the traces of the plane in which these lines lie. Measure in degrees the angle between the plane of the end posts and the top plane.

24. In practice, the problems of descriptive geometry are mainly those of finding the intersections of planes with other planes, or of lines with planes. An application of the methods of the preceding paragraphs to an actual problem is shown in Fig. 12. A horizontal triangular prism and a square pyramid are shown in their top and front projections. It is required to find the lines showing their intersection. The planes of the sides of the object will intersect in straight lines, called the *lines of intersection*. The intersection of the prism and the pyramid is made up of parts of the lines of intersection of the planes forming their sides. The first operation is to find the traces of the planes of the sides actually intersecting each other. These traces are determined by finding the intersections of two lines in each side with the top and front planes and joining the corresponding intersections. (Art. 23.) Evidently it is unnecessary to find the traces of the sides not intersecting, and those to be omitted can be usually determined by inspection. In this problem the traces of all the planes are easily found.

The top trace, TM , of the side $abcd$ passes through the point of intersection of the line ad with the top plane. This is the point ti^2 . This point alone, however, does not determine TM ; but, in this case, the projection of one line (ab) in the required plane is parallel to a plane of projection, and therefore the trace, TM , is parallel to the projection $a'b'$. We draw TM then through ti^2 and parallel to $a'b'$. The front trace of the same side is the line FM passing through the points of intersection of the lines ab and ad with the front plane. These intersections are marked respectively fi^1 and fi^2 . The traces of the side efg are then readily found. The top trace, TN , is drawn through the intersections of ef and eg with the top plane (ti^3 and ti^4); and the front trace, FN , through the intersection of ef with the front plane fi^3 . These three points determine both traces of the side efg . The front trace of eg was not found because the construction lines intersected off the sheet. To check the accuracy of the traces found, an additional intersection fi^5 , the intersection of fg with the front plane, was located. Where the top traces TM and TN of the sides $abcd$ and efg intersect, we find a point o' in the top plane common to the two planes of the sides. This point being in one of the planes of projection has its front projection in the XX axis at o' . This is one point in the required line of intersection. The intersection of the front traces of the

same sides is the point p' , the front projection of another point in the line of intersection. The top projection of this point is p' on the XX axis. The two projections of the line of intersection are found by joining the corresponding projections of the points o and p . A part of this line jk lies on the surface of both the prism and the pyramid, and is a portion of the required intersection of the two objects. The line of intersection of the planes $abcd$ and egm gives another part marked jl , and finally the intersection of the lower side of the prism with the sides of the pyramid completes the required intersection.

25. In a case where both planes are parallel to the XX axis, their line of intersection is likewise parallel to the axis; and it is determined by the intersection of the *side traces* of the given planes. In Fig. 13 the planes P and Q are represented by their top and front traces parallel to the XX axis. The side traces of the planes are shown at the right on the drawing, intersecting at the point x' . The top and front *projections* of the required line of intersection are shown by $x'x$ and $x'x'$.

26. PROBLEM 6. *To find the line of intersection of two planes.*

Method. The point where the top traces intersect is the point of intersection of the required line with the top plane. The point where the front traces intersect is the point of intersection of the line with the front plane. After locating the two projections of each of these points join the corresponding projections to obtain the required line of intersection.

Exercises

30. Find the line of intersection of two planes M and N . The top and front traces of the plane M intersect at $x = -3$, making angles respectively of 60° and 300° ; and the traces of the plane N intersect at $x = -1$, making angles of 135° and 210° .
31. Draw the top and front projections of the line of intersection of the plane of the end posts in Exercise 29 with another plane, parallel to the XX axis, making an angle of 30° with the top plane and passing through a point 1 inch behind the front plane and $\frac{1}{4}$ inch below the top plane. (Use the side plane.)

32. Find the projections of the intersection of the plane, M , in Exercise 30 with a plane, P , with its top and front traces intersecting at $x = -1$ and making angles respectively of 135° and 300° with the axis. (Observe that the line of intersection is parallel to the front plane and that the front projection is parallel to the front traces.)
33. A plane, Q , has its top and front traces intersecting at $x = 0$, and making angles respectively of 135° and 275° with the axis. Find the line of intersection of this plane with a vertical plane 1 inch behind the front plane. (The front projection is parallel to the front trace of Q . The top projection is parallel to the XX axis.)
34. Find the line of intersection of the plane, M , in Exercise 30 with the plane, Q , in Exercise 33. (Draw an auxiliary plane parallel to the front plane, cutting both planes. It will cut from each plane a line parallel to the front plane. The intersection of these two lines is one point in the line of intersection of the two planes.)
35. Take the line $a(-2\frac{1}{2}, 0, \frac{3}{4})$, $b(-2, 1, 0)$, as the line of intersection of two planes R and S . Through the front projection of a , draw the front traces of the two planes in any directions. Determine the top traces.
36. In Fig. 14 the plan and elevation of the roofs of a stable and an adjoining shed are shown. Complete the drawings by showing the projections of the line where the two roofs intersect. The dimensions and slopes necessary for drawing the traces of the roof planes are given.
27. In a complicated drawing it is often necessary to find the true size of a plane surface which is not parallel to any plane showing true dimensions. The usual process is to imagine the plane of the surface extended to intersect either the planes of projection, or a plane or planes parallel to them, thus determining the top and front traces of the plane in which the surface lies. About one of these traces as an axis, the plane is then revolved till it is parallel or coincides with the top or front planes.
- The simplest case of finding the true size of a plane surface is illustrated at the left-hand side of the roof shown in Fig. 15. The side abc is not shown in its true size, neither are its angles. If, however, the side is revolved about its intersection with the plane of

the base of the roof, $b'c'$, into the plane of the base, its true size will be seen. The lines ab and ac will be then seen in their true lengths and with the true size of the angle between them. In this revolution the path traced by the point a is shown in its front projection by the dotted lines of the arc of a circle with a centre at $b'c'$. The point a' shows its revolved position in the plane of the base, and $a'b'c'$ shows true dimensions for any part of the outline of the surface.

28. In the preceding case, the problem was simplified because the surface to be determined was perpendicular to the front plane of projection, so that the radius for the revolution of the corner a was shown in its true length in its front projection. At the right-hand side of Fig. 15 the side of the roof def is represented. When now the revolution is made about the line $e'f'$ (the intersection of the side with the plane of the base) as an axis, the point d moves in the arc of a circle with a radius equal to the perpendicular distance from d to the axis. The line $d'g$ measures this distance. Its true length is easily found from its two projections (Art. 20). The true length of this radius ($d'g'$) laid off on $d'g'$ produced shows the revolved position, d' , of the corner d . The true size of the given surface is shown by the triangle $d'e'f'$, from which true angles and lengths in the surface can be measured.

In Fig. 16 a perspective view of this roof is represented. It shows the right triangle $dd'g'$ (used in the preceding construction) in its actual position, together with the arc described by the point d in revolving to d' about $e'f'$ as an axis.

29. The right triangle $dd'g'$ (Fig. 15) shows in its revolved position also the true angle the side def makes with the plane of the base. This is the angle marked α in Figs. 15 and 16. It could have been found also by drawing a vertical plane perpendicular to $e'f'$, intersecting def and the base. The true angle between the lines of intersection of these planes is the same angle α .

30. Fig. 17 shows the simplest statement of the problem to find the true angle between two *intersecting lines** ab and cd . Before

* *Intersecting lines* have their corresponding projections intersecting each other in a point shown in both projections; that is, the intersection of the top projections of the lines must be in a vertical line through the intersection of their front projections. This is shown in Fig. 11 by the projections of the point a which is at the intersection of the lines ab and ac .

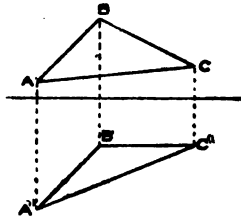
the lines can be revolved into one of the planes of projection the traces of the plane in which they lie must be determined. The intersections of the lines with the top and front planes are marked respectively ti and fi . The lines joining these points that are corresponding are the traces of the required plane. In this plane the lines must be revolved about one of the traces into either the top or front planes. Here the lines are shown revolved into the front plane about the front trace as an axis. The angle β is the true size of the acute angle between the given lines.

31. PROBLEM 7. *To find the true size of the angle between two intersecting lines.*

Method. Determine one of the traces of the plane in which the given lines lie. With this trace as an axis, revolve the two lines into the plane of projection in which the axis lies. The angle between the lines in their revolved position is the true size of the required angle.

Exercises

37. Find the true size of the obtuse angle between the two intersecting lines ab and cd in Exercise 21.
38. (a) It is required to find the angles between each of two guy ropes (ac and bc) and the mast (cd) of a derrick. These lines are shown in projection by the coördinates $a(-3, \frac{1}{4}, 1\frac{1}{2})$, $b(-2, \frac{1}{4}, 2\frac{1}{4})$, $c(-1, \frac{3}{4}, \frac{1}{4})$, and $d(-1, \frac{3}{4}, 1\frac{3}{4})$.
(b) Find the angle acb between the two guy ropes. Determine these angles by the simplest method applicable.
39. The vertices of the triangle shown in the figure below are at the points $a(-3, \frac{1}{4}, 1\frac{3}{4})$, $b(-2, 1\frac{1}{4}, \frac{3}{4})$, and $c(-\frac{1}{2}, \frac{1}{2}, \frac{3}{4})$. Find the true lengths of the sides ab and bc , and the projections of the bisector of the angle between these two lines.



40. A boat is towed along a canal by two mules, one walking on each bank. In projection drawing the boat is shown at $b(-2, \frac{1}{2}, 1)$. The tow paths are not at the same level and the positions of the mules are shown at $m_1(-1, 1, \frac{1}{2})$ and $m_2(-\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$. Find the true size of the angle between the tow ropes.

32. The angle which a line makes with its projection on any plane measures the true angle between the line and the plane. By drawing a line perpendicular to the plane from a point in the given line a right angled triangle is formed, in which the perpendicular is one leg, the projection of the line is another leg, and the line itself is the hypotenuse. The required angle is that between the hypotenuse and the leg lying in the plane; but the other acute angle of this triangle, the complement of the angle between the line and the plane, is more easily found by the method of Art. 31.

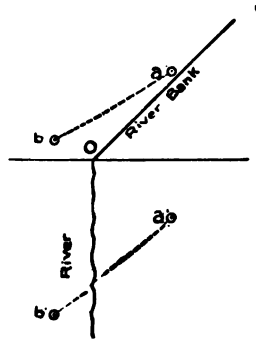
33. PROBLEM 8. *To find the angle between a given line and a given plane.*

Method. From any point in the given line draw a line perpendicular to the plane. Find the angle between these two lines. The complement of this angle is the angle between the line and the plane.

Exercises

41. The top and front traces of a plane intersect at $x = -1$, and make angles respectively of 135° and 210° with the XX axis. Find the angle a line $a(-1\frac{3}{4}, 1, 1)$, $b(-\frac{3}{4}, \frac{1}{2}, \frac{3}{4})$, makes with this plane.
42. Find the angle between a line with both projections parallel to the XX axis and the plane given in the preceding exercise.
43. Find the angle the line $c(-2, \frac{3}{4}, 1)$, $d(-1, \frac{1}{2}, \frac{3}{4})$, makes with the plane, M , with its top and front traces intersecting at $x = -3$ and making angles respectively of 135° and 240° with the XX axis. (Observe that the angle most conveniently found in this case is the supplement of the angle between the perpendicular and the given line of the right triangle mentioned in Art. 32.)

44. In constructing shadows on drawings we take rays of light that are reflected downward from the left-hand side, and shown by lines whose top and front projections make respectively angles of 135° and 225° with the XX axis. Find the angle between these rays and the top and front planes of projection.
45. A point a is given by its two projections. Through this point a line is to be drawn of which we know only the top projection and the angle it makes with the top plane. Draw the front projection.
46. The slope of a river bank makes an angle of 45° with the water-level. A horse on the bank draws a boat in the river. The traces on the horse are attached to a rope at a point $a(1, \frac{3}{4}, 1\frac{1}{8})$. The other end of the rope is fastened to the boat at $b(-\frac{1}{2}, 2, \frac{1}{4})$. Assuming there is no sag in the rope (that the points a and b in the figure below are in a straight line), find the true angle between the rope and the bank of the river.



34. When we wish to find the true angle between two sides of an object, the usual method is to pass an auxiliary plane perpendicular to both of the given sides. The intersections of this auxiliary plane with the planes of the sides will be lines showing the true angle between the planes. If one of the sides of the object is parallel to the plane of the drawing, as in the case of the side def of the roof in Figs. 15 and 16, the angle between this side and the plane of the base is found very simply, as explained in Art. 29. In the figures, α is the required angle.

The angle between the same side and the front plane cannot be found so easily by this method. On the other hand, however, when the traces of the planes to be dealt with are given or have been determined, all the ordinary cases are easily solved. If, for instance, a plane is given by its top and front traces, and the angle between it and the top and front planes is required, an auxiliary plane is used in the same general way as in the case above.

In Fig. 18 a plane, P , is given by its traces TP and FP . An auxiliary plane, M , is shown by the traces TM and FM , and is used here to find the angle between the given plane and the front plane of projection. It is, therefore, a plane which in space is perpendicular to the front plane and passes through any point in FP at right angles. As the auxiliary plane is perpendicular to the front plane, its top trace, TM , is perpendicular to the XX axis; and since it is passed perpendicular to the front trace of the plane P , its front trace, FM , is perpendicular to FP .

The true angle between the line of intersection of these two planes and the front plane is then the required angle between the given plane and the front plane of projection. The intersection is found by the method of Art. 26. The top traces intersect at b' and the front traces intersect at a' . Joining then the corresponding projections of the intersections, we obtain the line ab . The angle between this line and the front plane measures the angle between the plane, P , and the front plane. The point a is in the front plane. The point b is located the distance from b' to b' behind it. If the line ab is then revolved into the front plane about $a'b'$ as an axis, the point a remains stationary, and b revolves to the position b' , found by laying off $b'b'$ equal to $b'b'$, perpendicular to $a'b'$. The angle $b'a'b'$, marked θ , is the required angle between the plane, P , and the front plane of projection.

Similarly we can find the angle between the plane, P , and the top plane by drawing the top trace, TN , of an auxiliary plane, N , perpendicular to the top trace of P , and the front trace, FN , perpendicular to the XX axis. The required angle between the plane, P , and the top plane of projection is shown by the true size of the angle between the top plane and the line of intersection of the auxiliary plane N with the plane P .

35. The reverse of this last operation is made use of in drawing the traces of planes when only one is given on the drawing.

When the front trace of a plane is given and the angle the plane makes with the front plane is known, the top trace can be constructed. In Fig. 18, the front trace of the plane, P , and the angle θ the plane makes with the front plane are given. Draw the front trace, FM , of the auxiliary plane, and the line $a'b'$ making with $a'b'$ an angle equal to the angle the plane makes with the front plane. This last line is the revolved position of a line ab of which we know the front projection $a'b'$ and the distance ($b'b'$) of the point b behind the front plane. The top projection of b is located by laying off on a line perpendicular to the XX axis through b' the distance $b'b'$ equal to $b'b'$. The line ab determined now by two projections is a line in the plane and b' is a point in the top trace. The required top trace is drawn then through the projection b' and the intersection of the given front trace with the XX axis.

The method is exactly similar when the top trace and the angle the plane makes with the top plane are given, and the front trace is to be constructed.

36. PROBLEM 8. *To find the angle made by a given plane with either the top or front planes of projection.*

Method. To measure the inclination to the front plane: Through any point in the front trace of the given plane pass an auxiliary plane perpendicular to this trace. (It will be perpendicular to the front plane.) The true angle between the front plane and the line of intersection of the auxiliary plane with the given plane is the required angle.

To measure the inclination to the top plane: Pass an auxiliary plane perpendicular to the top trace. The true angle between the top plane and the line of intersection of the auxiliary plane with the given plane is the required angle.

Exercises

47. The top and front traces of a plane, P , make angles respectively of 135° and 210° with the XX axis. Find the angles the plane makes with the front and top planes of projection.
48. Given the traces of the plane, N (the side efg), in Fig. 12, find the angle between this plane and the plane of the base of the horizontal prism.

37. PROBLEM 9. *Given one trace and the angle a given plane makes with the corresponding plane of projection, to find the other trace.*

Method. Reverse the method of the preceding problem.

Exercises

49. The front trace of a plane, Q , makes an angle of 315° with the XX axis. The plane itself makes an angle of 45° with the front plane. Draw the top trace.
50. The top trace of a plane, R , makes an angle of 30° with the XX axis. The plane itself makes an angle of 45° with the top plane. Draw FR .

38. In Art. 29 the angle between two planes was found when one of the planes was parallel to a plane of projection. When, however, neither of the two given planes is parallel to the top or front planes, a different method must be used.

In Fig. 19 two planes, LMN and LNO , are shown in perspective, and we wish to determine the angle between them. If from any point p , we draw the lines pq and pr perpendicular respectively to the planes LMN and LNO , these perpendicular lines will determine a plane which is perpendicular to both of the given planes, and intersects them in the lines qe and er . The angle qer is therefore the angle between the two given planes. This angle is not usually determined directly. The method ordinarily used is to find the true angle qpr between the intersecting lines pq and pr (Art. 31). This angle is the supplement of the angle qer . Of course either of the angles $qer(\phi)$ or $qpr(180^\circ - \phi)$ is the angle between the two planes.

The above method is practically the same as finding the angle between the lines cut from the two given planes by an auxiliary plane passed through them, perpendicular to their line of intersection.

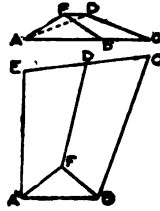
39. PROBLEM 11. *To find the angle between two oblique planes.*

Method. From any point in space draw a line perpendicular to each of the given planes. The angle between these two lines is the angle between the planes.

Exercises

51. Find the angle between the planes S and H in Fig. 14.
52. The slope of each of the sides of a square hip roof is 30° . Find the angle a line along a hip makes with a plane through the eaves.

53. The corners and ridge of an irregular hip roof are located by the following points: $a(0, 0, 5)$, $b(2\frac{1}{2}, 0, 5)$, $c(4, 0, \frac{1}{2})$, $d(2, \frac{3}{4}, \frac{3}{4})$, $e(0, 0, 1)$, and $f(1\frac{1}{2}, \frac{3}{4}, 4)$.



- (a) Find the actual angle between the sides of the roof intersecting at the ridge.
 (b) Find the angle of bevel of the top side of the hip rafters.
54. The *top* trace of a plane makes an angle of 45° with the XX axis, and the plane itself makes an angle of 60° with the *front* plane. Draw the front trace of the plane.

40. PROBLEM 12. *To draw through a given point a line parallel to a given plane.*

Method. Draw one projection of *any* line, and determine its other projection so that the line will lie in the given plane (Art. 15a). Through the two projections of the given point draw the projections of the required line parallel to the corresponding projections of the line drawn in the plane. The two lines shown now by parallel projections are parallel to each other; and the required line passing through the given point, being parallel to a line lying in the plane, is parallel to the plane.

Exercises

55. Through a point $a(-3, 1, 1\frac{1}{2})$ draw a line parallel to the plane, R , with its top and front traces intersecting at $x = -3$ and making angles respectively with the XX axis of 30° and 315° .
56. The top trace of a plane intersects the XX axis at $x = -1\frac{1}{2}$ and makes with it an angle of 120° . A point $o(-1, 1\frac{1}{2}, \frac{1}{2})$ lies in the plane. Determine the front trace of the plane.

41. Lines Perpendicular to a Plane. In Fig. 20 two planes, LMN and LNO , are shown. From any point p draw the lines pq and pr perpendicular to these planes. A plane which is perpendicular to both of the other planes is determined by these two lines, and intersects the planes in the lines de and ef . Assume now the plane LNO to be a plane of projection, on which LN is the trace of the plane LMN and the broken line qer is the projection of the lines pq and pr . Obviously LN is perpendicular to the plane pqr and therefore to the lines qe and er lying in this plane. The line er , however, is the projection of pq on the plane of projection and LN is the trace of the plane LMN . Therefore *when a line is perpendicular to a plane the top projection of the line is perpendicular to the top trace of the plane, and the front projection is perpendicular to the front trace.*

42. When one line is given by its projections and a plane is to be drawn perpendicular to it; from the last paragraph, it follows that the traces of the required plane must be drawn perpendicular to the corresponding projections of the line. In Figs. 21 and 22 a line ab is shown by its two projections and it is required to draw through a given point o a plane perpendicular to the line ab . Through the point o draw a line mo parallel to the top plane and which shall lie in the required plane. A line which is parallel to the top plane is commonly called a "horizontal," and is a line with its front projection parallel to the XX axis. And if this "horizontal" shall lie in the required plane, its top projection must be parallel to the top trace of the plane to be drawn. In other words, the conditions to be satisfied in this case are: (1) that the front projection $m'o'$ of the horizontal must be drawn parallel to the XX axis; and (2) that the top projection $m'o'$ must be drawn perpendicular to the top projection of the given line ab (the same as saying it must be parallel to the top trace of the required plane).

Having determined a line mo lying in the required plane, the front trace, FP , is drawn through the intersection (fi) of the line mo with the front plane and perpendicular to $a'b'$. The top trace, TP , is drawn through the intersection of FP with the XX axis and perpendicular to $a'b'$. A plane, P , perpendicular to the line ab and passing through a given point o , is thus determined.

A line parallel to the front plane and intersecting the top plane (determining the top trace first) could be used in the place of the one used here parallel to the top plane.

43. PROBLEM 13. *To pass a plane through a given point perpendicular to a given line.*

Method. Draw through the given point a line which will lie in the required plane (a "horizontal" line is usually most easily used). Through the intersection of this construction line with one of the planes of projection, draw one trace of the required plane perpendicular to the corresponding projection of the given line. The other trace is also drawn perpendicular to the corresponding projection of the line.

Exercises

57. Draw through the point $e(-1, 1, \frac{1}{2})$ a plane, U , which shall be perpendicular to the line $g(-3, 1\frac{1}{2}, 1)$, $h(-2, \frac{1}{2}, \frac{1}{2})$.
58. A plane with its top and front traces intersecting at $x = -2$ and making angles respectively of 45° and 330° with the XX axis is perpendicular to a line drawn through the point $j(-2, \frac{1}{2}, \frac{3}{4})$. Draw the projections of the line.
59. Through a point $g(2\frac{3}{4}, 1, 2\frac{1}{2})$ draw the projections of a line perpendicular to the plane of the right-hand side of the roof given in Exercise 53.
60. Show plainly the projections of an arrow at any point of the side of the roof mentioned in the preceding exercise, that will show a force due to wind pressure on a section of the roof. (Pressures of fluids are normal to the planes on which they are exerted.)
61. Draw the projections of the axis of a pipe which passes through the inclined floor of Exercise 13 at right angles and is joined at its intersection with the floor to the pipe mentioned in that exercise.

44. It is often necessary to find the point where a line intersects a plane. In Fig. 23 a line cd is shown by its projections and the plane, Q , by its traces. Evidently the line does not intersect the plane at the intersection of either projection with the corresponding trace, and a roundabout method must be employed for the solution.

If another plane, A , is passed through the line cd and perpendicular to the front trace, FQ , its front trace, FA , will coincide with the front projection of cd ; and because the plane, A , is made perpendicular to the front plane, its top trace, TA , will be perpen-

dicular to the XX axis. This auxiliary plane cuts a line mn from the given plane, Q (Art. 26). Since the line cd lies in the plane, A , it will cut the given plane in this line of intersection. Then where the top projection of the given line ($c'd'$) intersects the top projection of the line of intersection ($m'n'$) is the top projection (i') of the point where the given line intersects the given plane. The front projection of this point (i') is at the intersection with $c'd'$ of a vertical projecting line through i' .

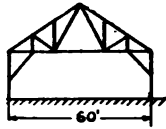
45. PROBLEM 14. *To find the point in which a given line intersects a given plane.*

Method. Pass any plane through the given line. (Usually a plane perpendicular to the front plane.) The line of intersection of this auxiliary plane with the given plane (Art. 26) intersects the given line at the point where the line intersects the plane.

Exercises

62. Using the results of Exercise 57, find the point where the line gh intersects the plane, U .
63. Find the point where the line along the ridge of a dormer window on the left-hand side of the roof given in Exercise 53 intersects the plane of the roof. The ridge is horizontal and is located by a point $h(\frac{1}{2}, \frac{1}{2}, 2)$ in it.
64. A steam pipe passes through a sloping floor at right angles. The axis of the pipe is located by the points $v(-2, \frac{1}{4}, \frac{1}{2})$ and $w(-\frac{1}{2}, \frac{3}{4}, 1\frac{1}{2})$. The plane of the floor is given by a vertical trace in a plan drawing, which intersects the XX axis at the origin taken for the coördinates. The floor makes an angle of 15° with the horizontal. Find the intersection of the axis with the plane.
65. Through the point h in the ridge of the dormer window in Exercise 63, draw the projections of a ray of light according to the conventional method used in Exercise 44. Locate the shadow of the ridge on the roof by finding the intersection of this ray with the plane of the side of the roof.

71. Find the shortest distance from the point h in the ridge of the dormer in Exercise 63 to the plane of the roof.
72. In Exercise 66 find the distance from the point L to the plane of the top of the desk.
73. In a roof of triangular cross-section, as shown in the figure, the ratio of the rise to the span is 1 to 4. A shaft is to be set up with its axis passing through a point 8 ft. above, and 25 ft. to the right of the eaves on the left-hand side. At this point it is supported by a hanger which is fastened to the roof at its other end at a point 5 ft. to the left of the point given and 2 ft. above it. The shaft is in a plane parallel to the section shown, and a plan drawing shows the distance between the vertical planes, through the point of attachment to the roof and the plane of the shaft, is 3 ft. Find the length of that part of the hanger included between these two points.



47. PROBLEM 16. *To project a given line upon a given plane.*

Method. Select any two points in the line and through each draw the projections of a line perpendicular to the plane. The line joining the points where these perpendicular lines intersect the given plane is the projection of the line upon that plane.

Exercises

74. In Exercise 43 project the line cd upon the given plane.
75. Project the triangle given in Exercise 39 upon a plane with its traces intersecting at $x = -3$. Both the top and front traces make angles of 10° with the XX axis, forming an acute angle between them.

48. PROBLEM 17. *To find the true distance from a given point to a given line.*

Method. Determine the plane of the point and the line by drawing through the point a line parallel to or intersecting the given line, and finding the intersections of both lines with either the top or the front plane. With a trace of this plane as an axis, revolve these lines into the corresponding plane of projection, and measure the distance required.

Exercises

76. Find the distance from the point $i(-2, 1, 1\frac{1}{2})$ to the line $j(-2, \frac{1}{2}, \frac{3}{4}), k(-\frac{1}{2}, 1\frac{1}{2}, \frac{1}{4})$.
77. The telephone wires running from a village to a house on the side of a mountain follow the shortest line between the points $v(-\frac{1}{2}, 1\frac{1}{2}, 2)$ and $h(-3, 0, \frac{3}{4})$. A camp at $c(-2\frac{1}{2}, 1\frac{1}{2}, \frac{1}{4})$ is to be connected to the telephone system of the village by erecting a line joining the one between v and h . Draw the projections of the shortest line that can be put up; and find the length of the line thus located.
78. In the triangle given in Exercise 39 find the distance from the point c to the line ab .

49. PROBLEM 18. *To find the shortest distance between two lines not in the same plane.*

Method. Two lines ab and cd not in the same plane are shown in Fig. 24. Pass a plane, MN , through ab parallel to cd (Art. 22). Project cd upon this plane. At the point e' where the projection $c'd'$ intersects ab , draw a line perpendicular to the plane intersecting cd at e . The true length of the perpendicular $e'e$ is the required distance.

Exercises

79. Find the shortest distance between the lines kl and mn in Exercise 27.
80. Find the distance between the guy ropes for the derrick in Exercise 38.
81. Through each of two lines pass a plane parallel to the other line. Find the shortest distance between these two planes.
82. Solve Exercise 70 by the method of this article.

50. PROBLEM 19. *To find the two traces of a plane when the angles the plane makes with both the top and front planes are given.*

Analysis. Imagine a sphere placed tangent to the plane of which we wish to find the traces, and two auxiliary planes are passed through the center of the sphere, one perpendicular to the top plane, and the other perpendicular to the front plane. These planes will each cut a circle from the sphere and a tangent line from the given plane. The angles these tangents make with the front and top planes respectively, are the angles the given plane makes with the front and top planes of projection.

Method. With any convenient point as a center and any radius describe a circle representing the revolved position of the circles cut from the sphere by both of the auxiliary planes. Draw a line in the front plane tangent to the circle, making with a radius prolonged an angle equal to the angle the given plane makes with the front plane. This tangent is then the revolved position in the front plane of the line of intersection of this auxiliary plane with the given plane (Art. 36). One end of the line of intersection showing the angle with the front plane is a point on the front trace of the required plane. The other line of intersection is used in the same way. Revolve the lines of intersection back to the original positions of the auxiliary planes. Through the points stationary in revolution draw the traces of the required plane parallel to lines tangent to the circle at the points cut by the radii used in the construction, and through the revolving end of the other line of intersection as located after counter revolution.

Exercises

83. Draw the traces of a plane which is inclined 30° to the top plane and 45° to the front plane.
84. Draw a plane which makes angles of 45° and 60° respectively with the top and front planes.

51. PROBLEM 20. *Given the diameter and the projections of the centre of a circle, to draw the projections of the circle in a given plane.*

Method. The centre of a circle is shown in Fig. 25 at the point e in the plane, P ; and the top and front projections of the circle are to be drawn that it shall also lie in the plane. Revolve the centre of the circle about FP as an axis into the front plane, and show the true size of the circle. When this circle is revolved back into the plane, P , both projections will be ellipses. The length of the major axis of the ellipse in the front projection is the same as the diameter of the circle. As all lengths which are perpendicular to the axis of revolution are fore-shortened in the front projection, the minor axis is the fore-shortened diameter at right angles to FP . Draw through the centre c a line in the plane, P , parallel to the front plane. Through its intersection with the top plane at o and perpendicular to the trace FP draw an auxiliary plane, A . Its line of intersection with the given plane is equal in length to the shortest distance from c to FP . Revolve this line into the front plane to show its true length and from the end of this line at o' , lay off a length r equal to the radius of the circle. This determines the length of the semi-minor axis $b'c'$ of the elliptical front projection of the circle. The top projection of the circle could be constructed by repeating the same process, or by drawing the top projections of lines tangent to the front projection.

Exercises

85. Draw the projections of a circle, diameter $1\frac{1}{2}$ inches, lying in the plane given in Exercise 55.
86. Two pulleys rotating on lines of shafting at right angles to each other are to be connected by a belt running over a guide pulley. Determine the plane of the intermediate pulley to properly direct the belt, drawing also the projections of all the pulleys.

CHAPTER III

PROBLEMS RELATING TO PLANES TANGENT TO SOLIDS

52. Every surface may be generated by the motion of a line; and the different positions assumed by this line are called the *elements* of the surface.

53. A *surface of revolution* is one that is generated by the revolution of a given line about a straight line in its plane, as an axis.

A *single curved surface of revolution*, such as a cylinder or a cone, is one in which the elements are straight lines; and any two consecutive elements are in the same plane.

A *double curved surface of revolution*, such as a sphere, ellipsoid, torus, etc., is one which contains no elements that are straight lines.

54. PROBLEM 21. *Given one projection of a point on the surface of a cylinder to find its other projection.*

Method. Through the given projection of the point draw an element of the surface parallel to any element shown. This will be the projection of two elements on opposite sides. Determine the corresponding projections of the intersections of these elements with the base. Draw the two projections of the elements and project the given point on them. (Cf. Fig. 26.)

55. PROBLEM 22. *Given one projection of a point on the surface of a cone to find the other projection.*

Method. Same as the preceding except that the element drawn through the point passes through the vertex of the cone.

56. PROBLEM 23. *To pass a plane tangent to a cylinder, or a cone, through a given point on the surface.*

Method. Through the given point draw an element of the surface (Arts. 54, 55). At the intersection of this element with the base, draw a line tangent to the base. The required plane is determined by the element and this tangent line.

Exercises

87. Pass a plane tangent to an oblique cylinder through a point *b* on its surface.
88. The base of an inverted right circular cone is in the top plane, and the angle between the elements of the surface and the axis is 30° . Pass a plane tangent to the cone through a point *c* on its surface.

57. PROBLEM 24. *To pass a plane tangent to a cylinder, or a cone, through a given point without the surface.*

Method. Through the given point draw the projection of a line (parallel to the element in the case of the cylinder and through the vertex for the cone), producing it to intersect the plane of the base. Draw a tangent to the base through this point. The required plane is determined by this tangent line and the line through the given point.

Exercises

89. Pass a plane tangent to an oblique cone through a point in the *XX* axis.
90. Pass a plane tangent to a cylinder which has one base in the top plane and another in the front plane, and *through* a point in the front plane outside the base.

58. PROBLEM 25. *To pass a plane tangent to a cylinder and parallel to a given line.*

Method. Through any point in the given line draw a line parallel to an element of the cylinder. The plane determined by these two lines will be parallel to the required plane.

Exercises

91. Pass a plane tangent to an oblique cylinder and parallel to the *XX* axis.
92. Pass a plane tangent to a right circular cylinder with its base in an oblique plane, and parallel to a line in the plane.

59. PROBLEM 26. *To pass a plane tangent to a cone and parallel to a given line.*

Method. Through the vertex of the cone draw a line parallel to the given line. These two lines determine the required plane.

Exercises

- 93. Pass a plane tangent to an inverted oblique cone and parallel to an oblique line.
- 94. Pass a plane tangent to a right circular cone and parallel to a line in the side plane.

60. PROBLEM 27. *To pass a plane tangent to a sphere through a given line.*

Method. Pass an auxiliary plane perpendicular to the given line through the centre of the sphere. (The plane cuts a great circle from the sphere and a line tangent to this circle from the required tangent plane.) Revolve the auxiliary plane, with its intersections with the given line and the sphere into a plane of projection. Through the point of intersection draw a line tangent to the circular intersection with the sphere. A plane passed through the tangent line (in its true relative position) and the given line is the one required.

Exercises

- 95. Pass a plane tangent to a sphere passing through any oblique line.
- 96. Pass a plane tangent to a sphere with its centre in the top plane and passing through a line parallel to the XX axis.

CHAPTER IV

INTERSECTIONS AND DEVELOPMENTS OF SOLIDS

61. The intersections of solids are found by locating corresponding lines in each surface, and joining the points where these lines intersect. These points of intersection are points on both solids and are therefore points in their line of intersection.

62. A plane which is tangent to a single curved surface (Art. 53) contains an element of the surface. If this surface is rolled on a tangent plane until each of its elements has come into this plane, the part of the plane passed over, and included between the extreme elements is a plane surface equal to the given surface. The surface thus passed over is called a *development*. In order to determine the positions of the different elements of the surface as they come into the plane of development, it is necessary to locate some curve upon the surface which will develop into a straight line or a simple curve, upon which the actual distances between the elements can be laid off.

In Fig. 27 an oblique circular cylinder is shown. Through the point *a* a plane perpendicular to the axis is passed, cutting the surface in the line through *ae*. The curve of intersection is a circle which when revolved through 90° into the plane of the drawing is shown as the circle *abd*. The circumference of this circle is the actual length of the development of the cylinder.

If we wish to develop the part of the cylinder above the intersection, we draw the base line *aa*, making its length equal to the circumference of the circular intersection. A point on the curve of the development is immediately located over the point *a*, at a distance laid off on an element, equal to that from *a* to the top base. Other points are taken in the same way. To locate a point *e*, the distance *ae* is laid off on the base line, equal to the arc *ab*, and the length *ed* is the width at this point, of the surface we are developing. By continuing the process a series of points is obtained, and the surface included between the curve drawn through these points and the base line is the development of the surface of the cylinder above the intersection with the plane passed through *a*.

63. PROBLEM 28. *To find the intersection of any cylinder with any plane.*

Method. Pass through the cylinder a series of auxiliary planes, taken either perpendicular to its axis or parallel to its axis and perpendicular to one of the planes of projection. For the curve of intersection join the points where the lines cut from the cylinder intersect the lines cut from the given plane.

Exercises

97. Find the shape of the hole cut for a circular chimney in a tin covering for the right-hand side of the hip roof given in Exercise 53.

98. Draw the pattern for the vertical end of a bath-tub with a semi-circular cross-section.

64. PROBLEM 29. *To find the intersection of any cone with any plane.*

Method. Pass through the cone a series of auxiliary planes, taken either perpendicular to its axis or through the vertex and perpendicular to one of the planes of projection. For the curve of intersection, join the points where the lines cut from the cone intersect the lines cut from the plane.

Exercises

99. Taking the data of Exercise 66 determine the curve of intersection of the cone of rays of maximum intensity with the plane of the desk.

100. Draw the curve of intersection of a right cone with an oblique plane.

65. PROBLEM 30. *To find the line of intersection of any surface of revolution with any plane.*

Method. Pass through the surface of revolution a series of auxiliary planes perpendicular to its axis. These planes cut circles from the surface of revolution. The intersections of the circles cut from the surface and the lines cut from the plane determine the required curve of intersection.

Exercises

101. Find the intersection of a torus with an oblique plane.
102. Draw the projections of a hexagonal nut, showing accurately the lines of intersection.
103. Draw accurately the curve on the "stub end" of a connecting rod (cf. Fig. 32).
104. The blades of a ventilating fan are plane surfaces attached to a spherical hub. Show the intersections.

66. PROBLEM 31. *To find the curve of intersection of two cylinders.*

Method. Pass through the cylinders auxiliary planes parallel to both axes. The elements cut from each cylinder by these planes determine by their intersections the required curve.

Exercises

105. Draw accurately the lines of intersection appearing on the surface of a flanged pipe fitting shown in Fig. 29. The fitting is made up of two cylinders with their axes intersecting each other symmetrically. The necessary dimensions are shown.
106. Find the intersection of two oblique cylinders with their bases in the top plane.
107. A vertical steam drum is to be put on a horizontal cylindrical boiler. Make a pattern to show the size of the hole to be cut into the boiler plate.

67. PROBLEM 32. *To find the curve of intersection of two cones.*

Method. Pass through the cones auxiliary planes drawn through the line joining their vertices. Elements cut from each cone by these planes determine by their intersections the required curve.

Exercises

108. Find the curve of intersection of two right cones with axes that do not intersect. The axis of one cone is horizontal, the other is vertical. Draw the development of the cone with the horizontal axis, showing the curve of intersection.
109. Draw the curve of intersection of two oblique cones with intersecting axes. Show the development of one of the cones.

68. PROBLEM 33. *To find the curve of intersection of a cylinder and a cone.*

Method. Pass through the vertex of the cone auxiliary planes parallel to the axis of the cylinder. The elements cut from each surface by these planes determine by their intersections the required curve.

When the two surfaces have circular bases, it is most advantageous to use auxiliary planes which cut circles from each surface. Fig. 28 illustrates this method of finding the intersection of an oblique cylinder and a right cone. The auxiliary planes are shown by horizontal lines in the front view and by circles on the two surfaces in the top view. The intersections of the corresponding circles determine the curve of intersection.

Exercises

110. Find the intersection of an oblique cone with an oblique cylinder. The axes do not intersect, and one axis is perpendicular and the other parallel to the front plane.
111. A circular tower has a conical roof through which a horizontal pipe passes. Find the size of a covering for the roof and show in the development the hole cut for the pipe.
112. Make the necessary patterns for an arch stone of a conical arch in a circular wall.

69. PROBLEM 34. *To find the line of intersection of any two surfaces of revolution (sphere, ellipsoid, cone, etc.).*

Method. If the axes of the two surfaces intersect, the point of intersection of the axes is taken as the centre for a series of auxiliary spheres, cutting circles from each surface. The intersections of these circles with each other are points on the required curve.

If the axes do not intersect, pass through the surfaces auxiliary planes perpendicular to one axis and cutting circles from one surface and some other curve from the other surface. The intersections of the corresponding curves give the points to be found.

Fig. 29 shows intersection of a sphere with an ellipsoid. The point *m* is taken as the centre for the auxiliary spheres (at the intersection of axes). The intersections of the circles cut from each surface give the points on the curve of intersection *a'b'*. Auxiliary

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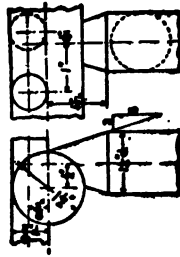


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CHAPTER V

MISCELLANEOUS EXERCISES

116. A vertical steam pipe, $1\frac{1}{2}$ inches diameter, is intersected by a conical nozzle and two smaller pipes as shown in the figure. The slope of the curved surface of the nozzle is $\frac{3}{10}$.



- (a) Determine the developed true size of the hole cut into the vertical pipe by the nozzle.
 - (b) Develop the conical surface of the nozzle between its intersections with the vertical pipe on the left and the horizontal pipe on the right.
117. A cylindrical pipe, 2 ft. diameter, passes through the roofs shown in Fig. 14 at a point in the intersection of the roofs. The axis of the pipe is perpendicular to the line of intersection and is inclined 45° from the vertical. Find the true size of the hole cut out of the roof for the pipe.
118. Obtain by development the true size and shape of the covering needed for a symmetrical dome with eight sides.
119. Find the shadow of a sphere, (a) on a plane below and parallel to the top plane; (b) on an oblique plane.
120. Develop the conical portion of a double elbow connecting two circular pipes of different diameters.
121. Make the drawings for the patterns of a bath-tub (cf. Fig. 88).



122. Find the intersection of a sphere with a cylinder whose axis does not pass through its center.
123. Find the intersection of the cylindrical ceiling of a corridor with the hemispherical ceiling of a vault.
124. A semi-circular wire loop, $1\frac{1}{2}$ inches diameter, rotates on a vertical axis supported at the points $a(-1, \frac{1}{2}, \frac{1}{4})$ and $b(-1, \frac{1}{2}, 1\frac{3}{4})$, at the ends of its vertical diameter. A ball, $\frac{1}{2}$ inch diameter, is attached to the end of a horizontal supporting arm, and revolves about the point $c(-2\frac{1}{2}, 1\frac{1}{2}, \frac{1}{2})$. The distance from the center of the ball to the center of revolution is 1 inch. Will the ball meet the wire loop?
125. A metal shade for an electric lamp is made up of a hemisphere and half of a circular cylinder. The axis of the cylinder passes through the center of the sphere. Draw the intersection of the two surfaces, and a pattern to be used in cutting out the metal to make the shade.
126. Find the intersection of a hexagonal prism with an oblique plane. Draw the development of the part of the prism above the line of intersection.
127. A shaft of a mine follows approximately the line between the points $a(-3, \frac{1}{2}, 1\frac{1}{2})$ and $b(-1, 1\frac{1}{2}, \frac{1}{8})$. A tunnel is to be made from a point $c(-2\frac{1}{2}, 1\frac{1}{2}, \frac{1}{2})$ in the side of a mountain to intersect the shaft. Find the shortest length of the tunnel, and the angle made by the center lines of the shaft and the tunnel with each other.

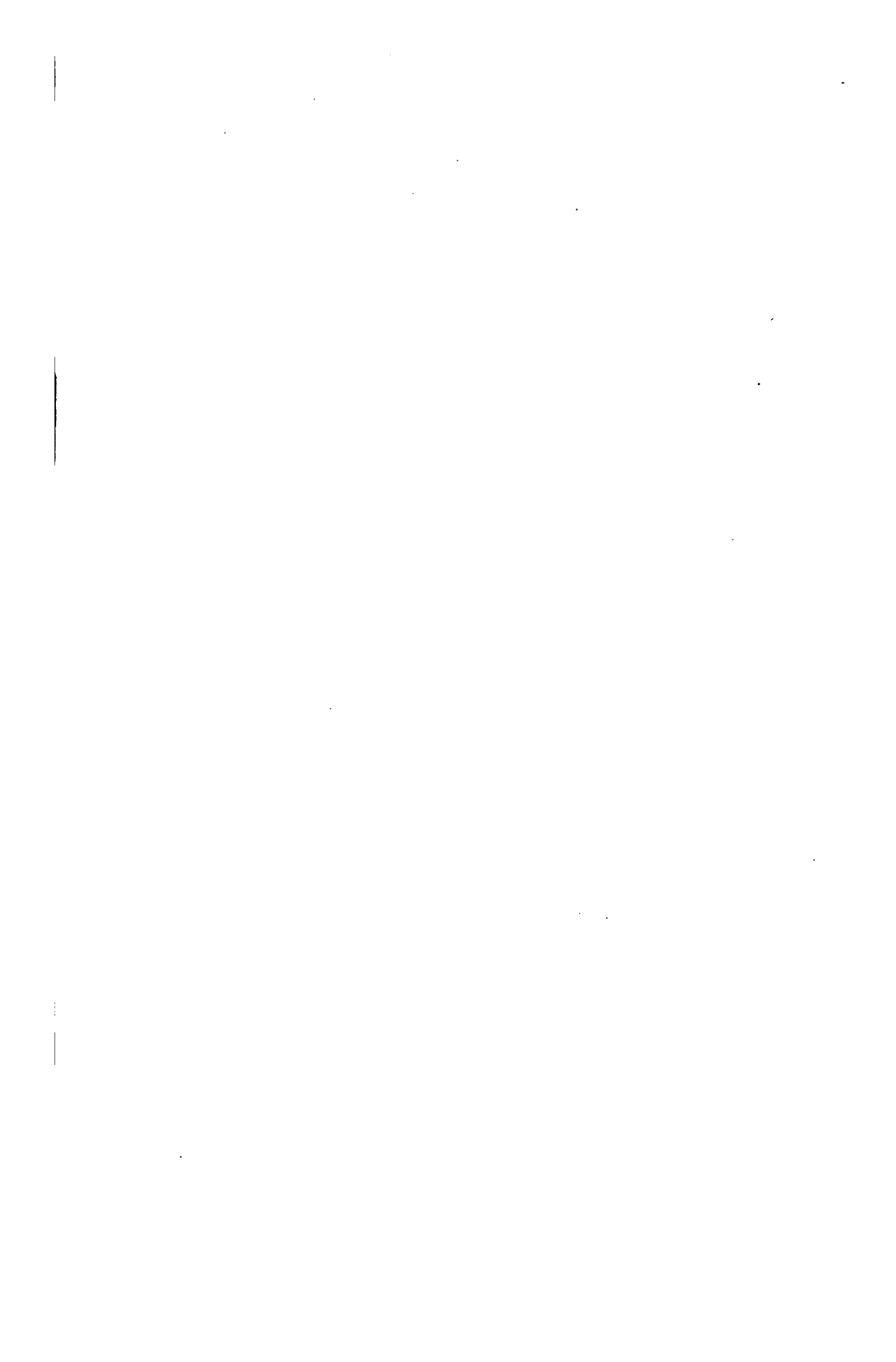




Fig. 9

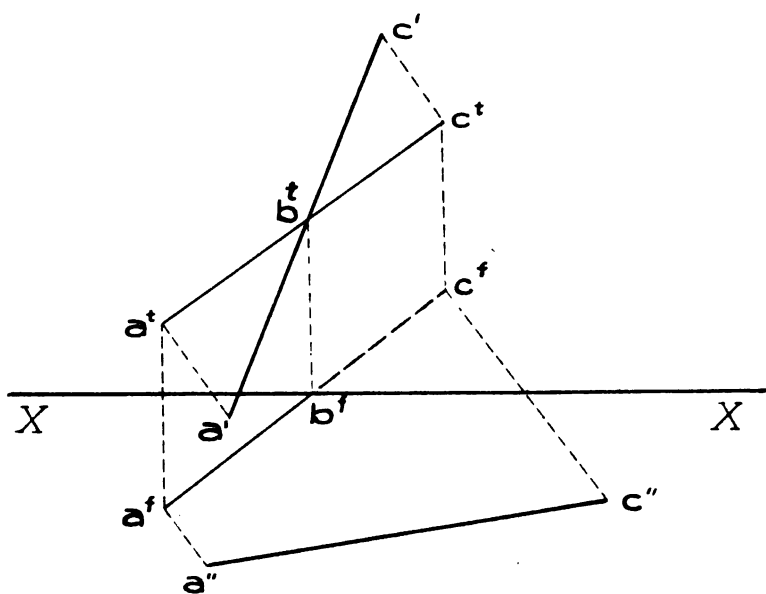


Fig.10

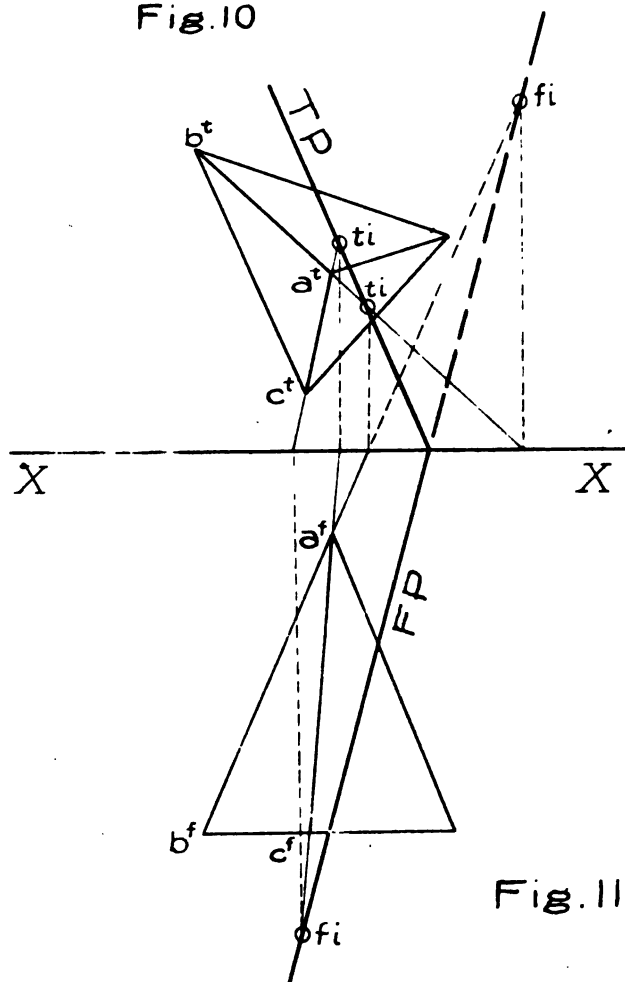
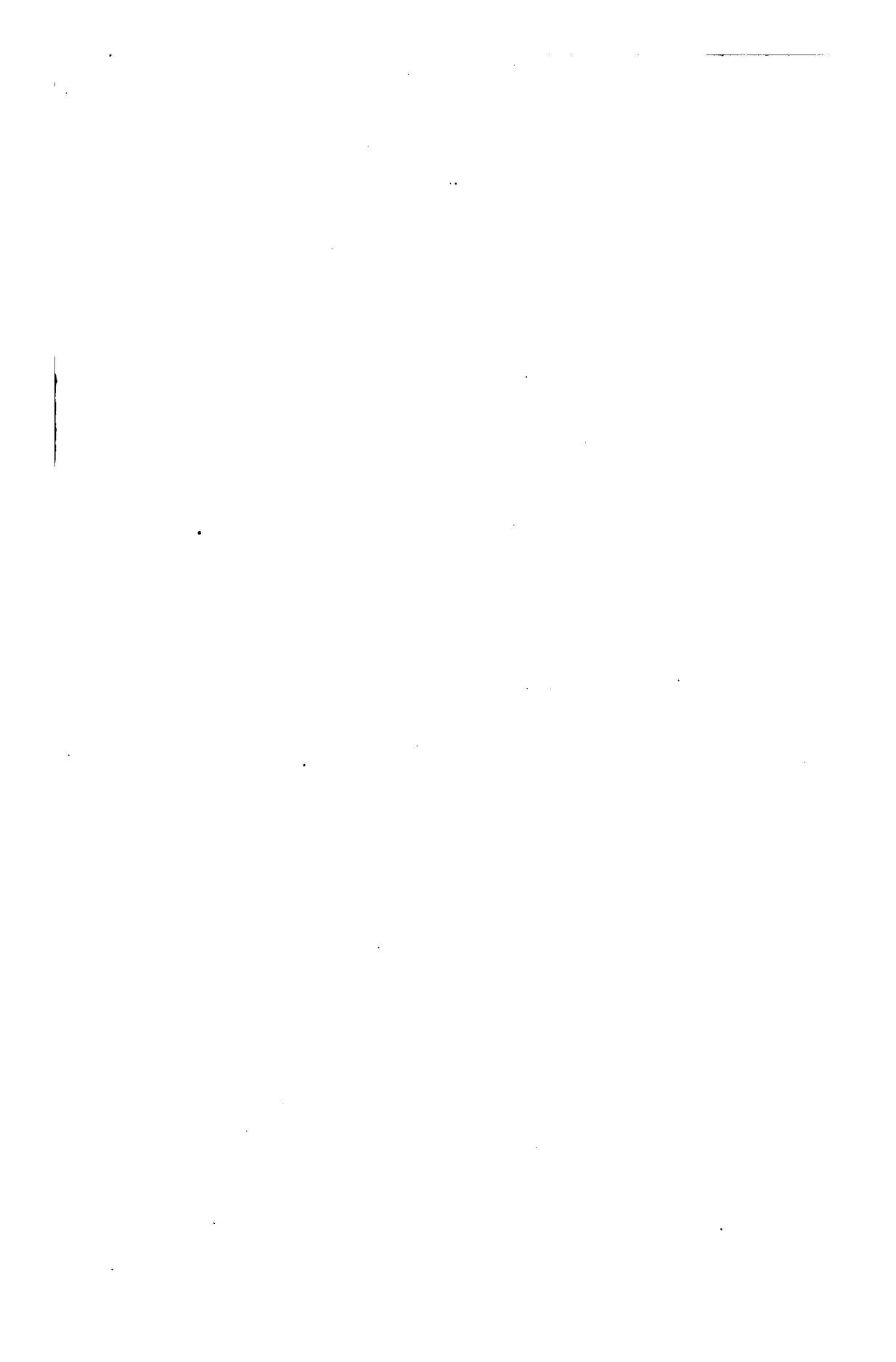
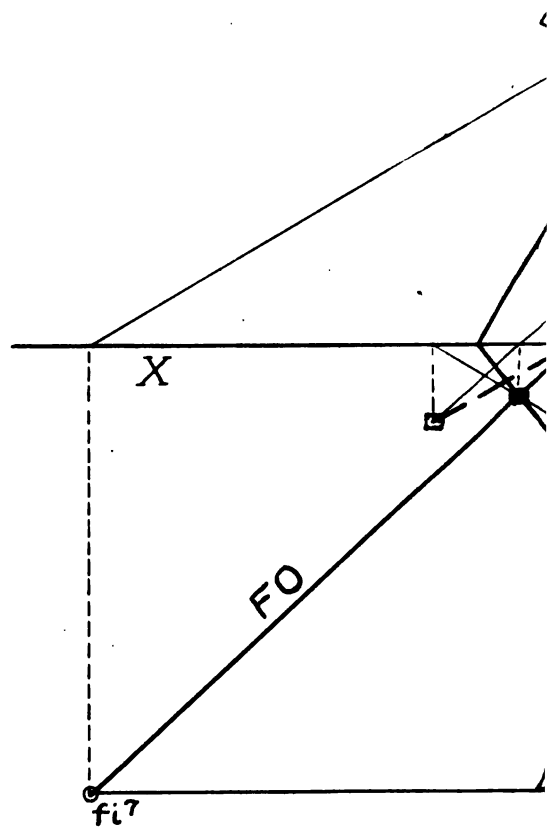
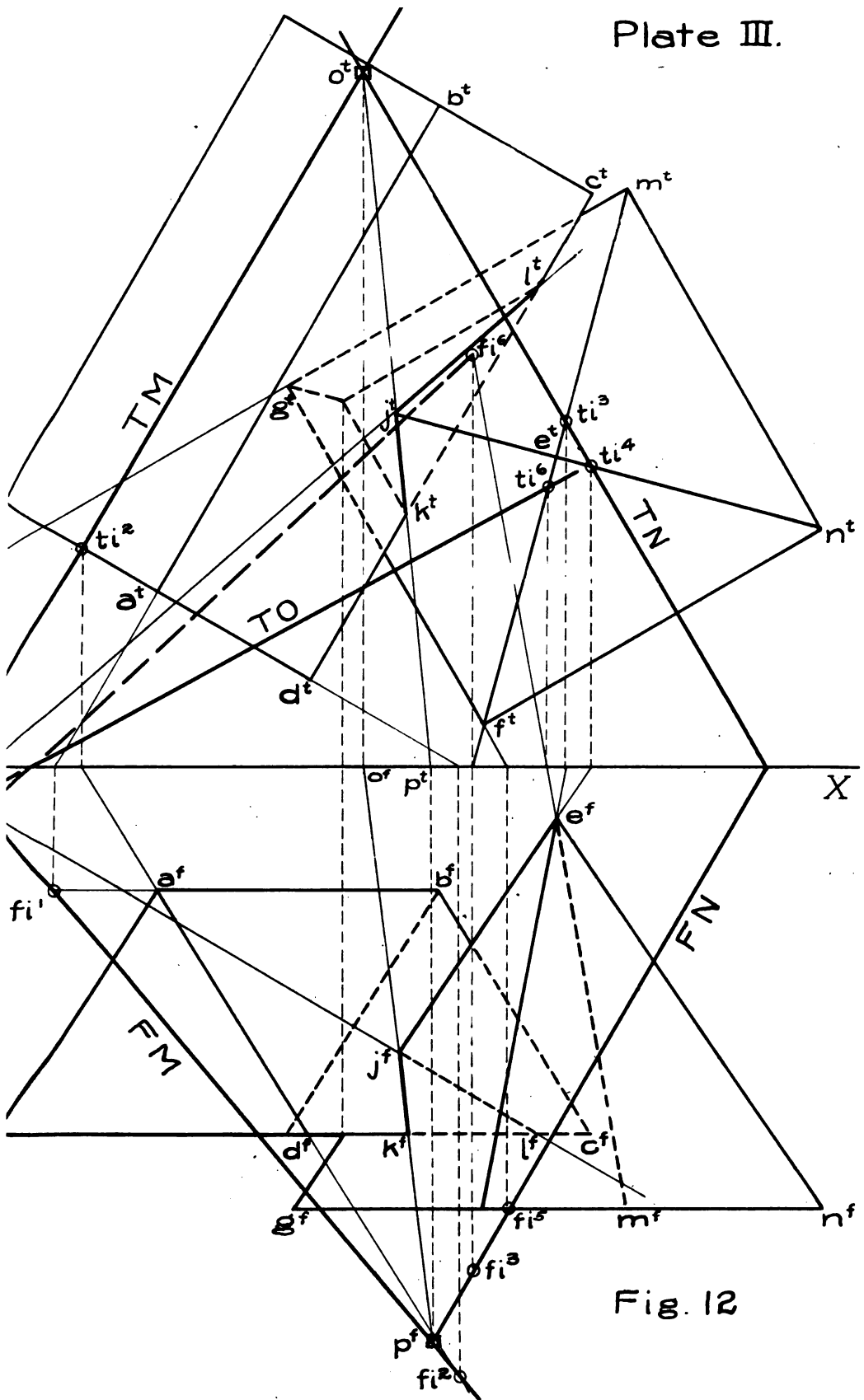
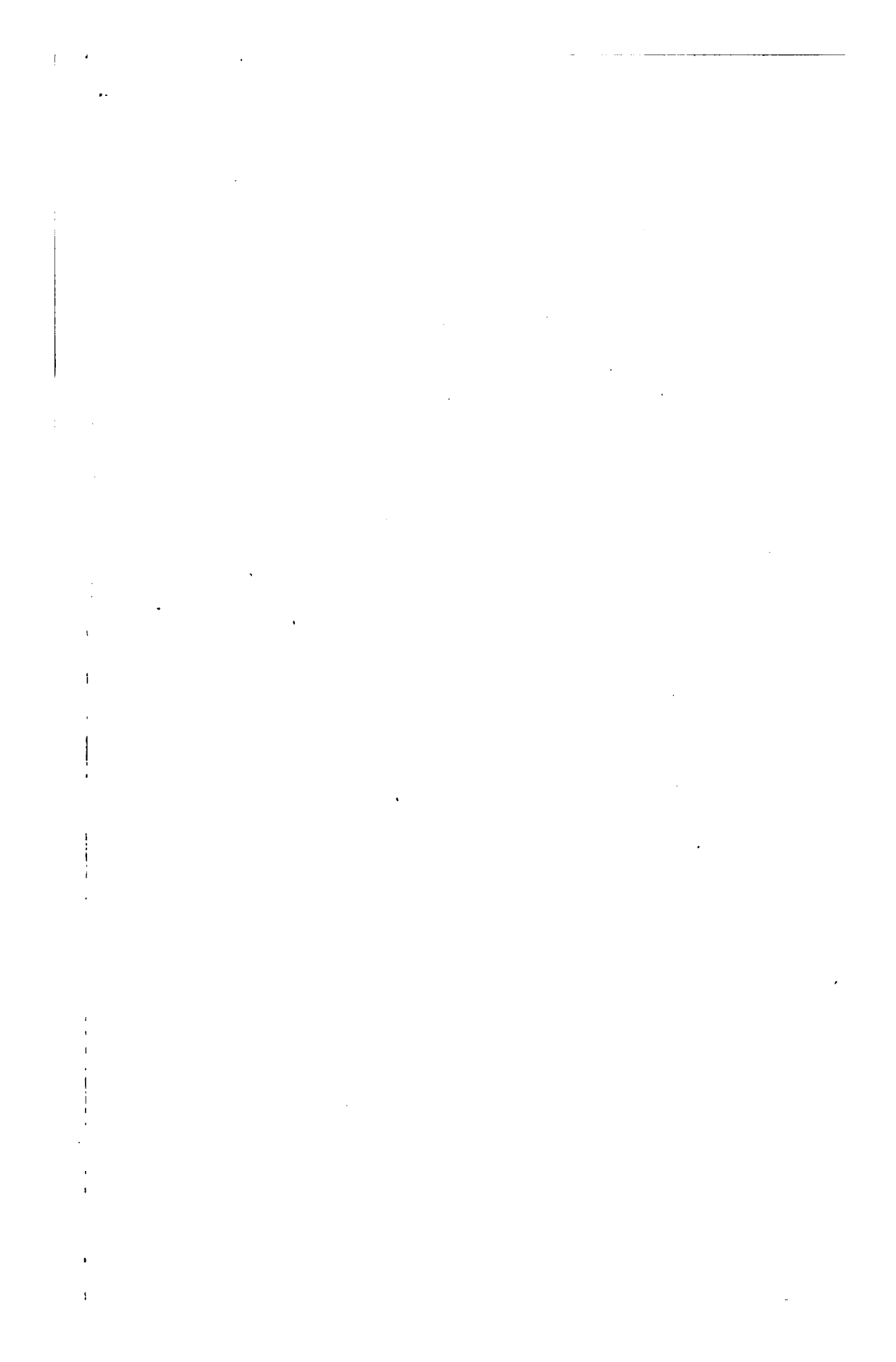


Fig.11









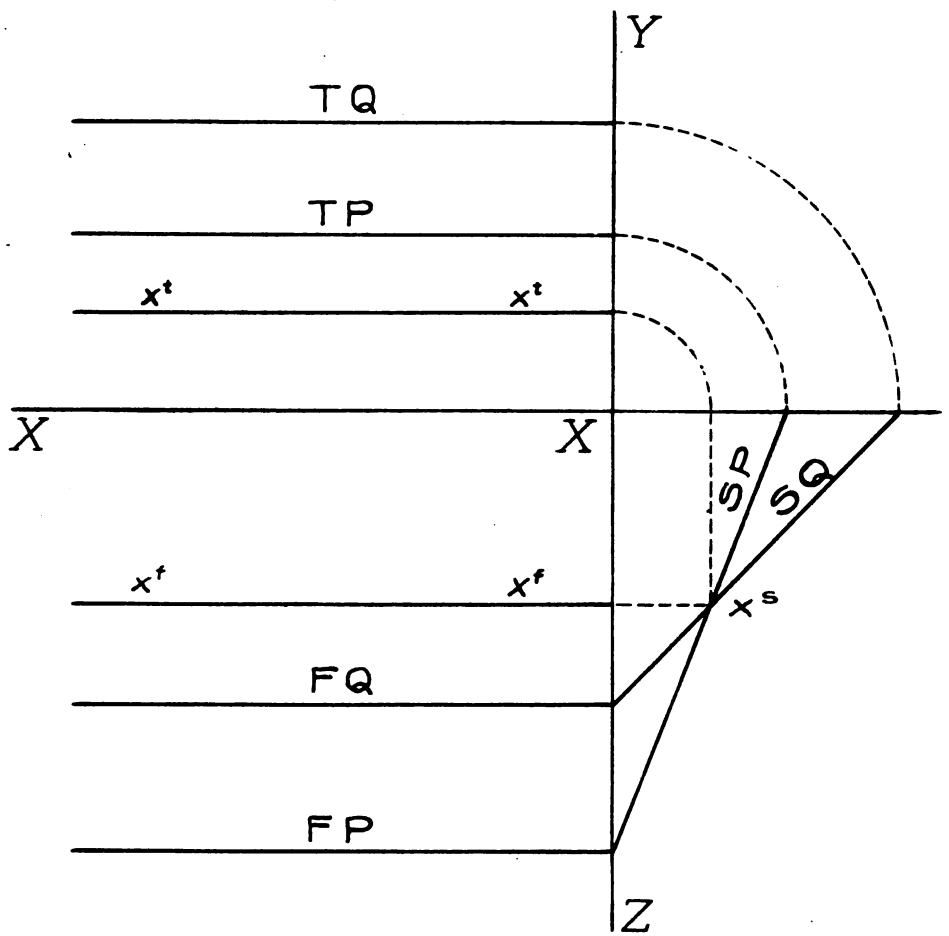
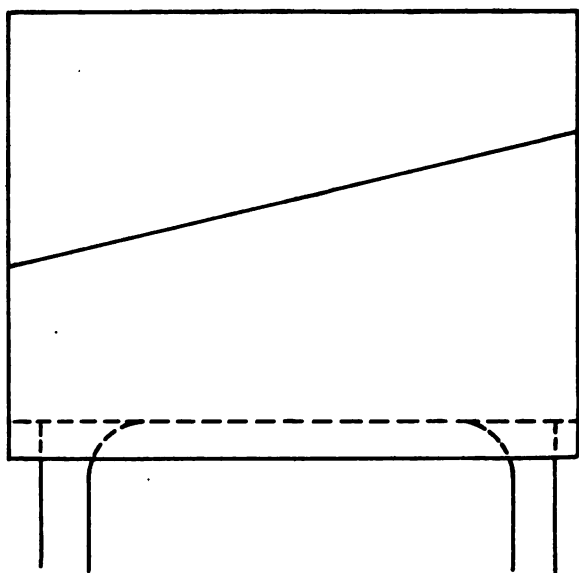


Fig. 13.



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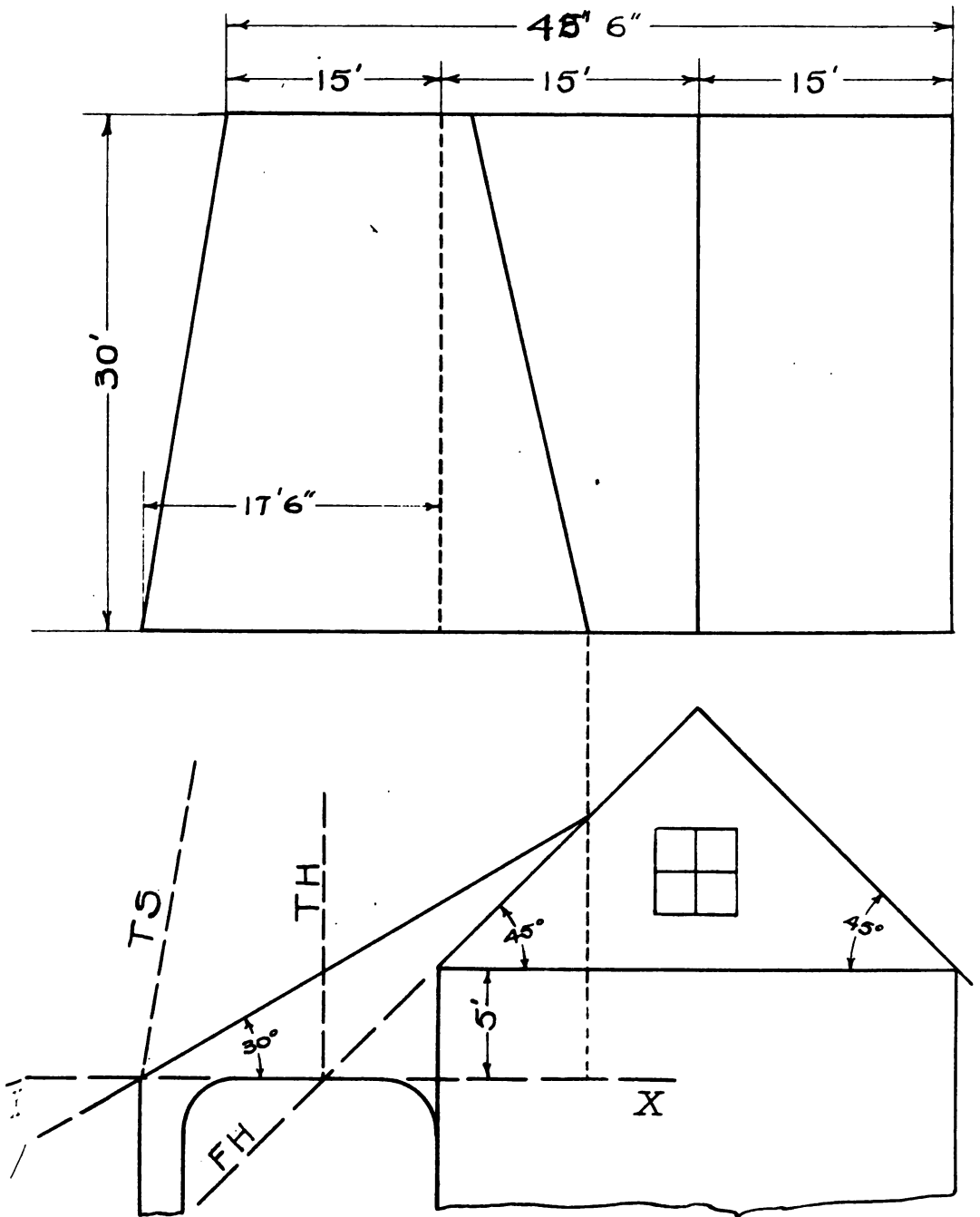
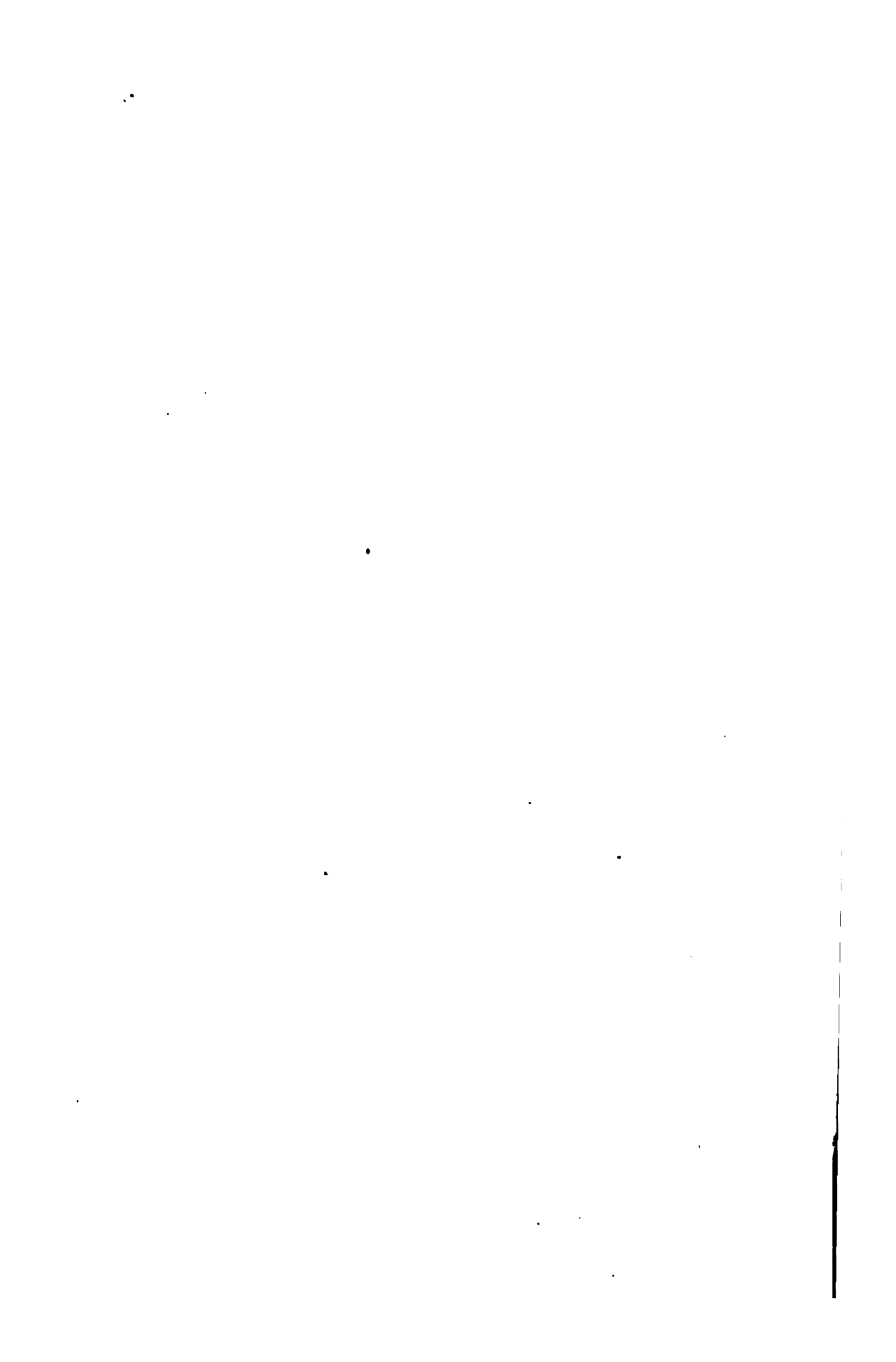


Fig. 14



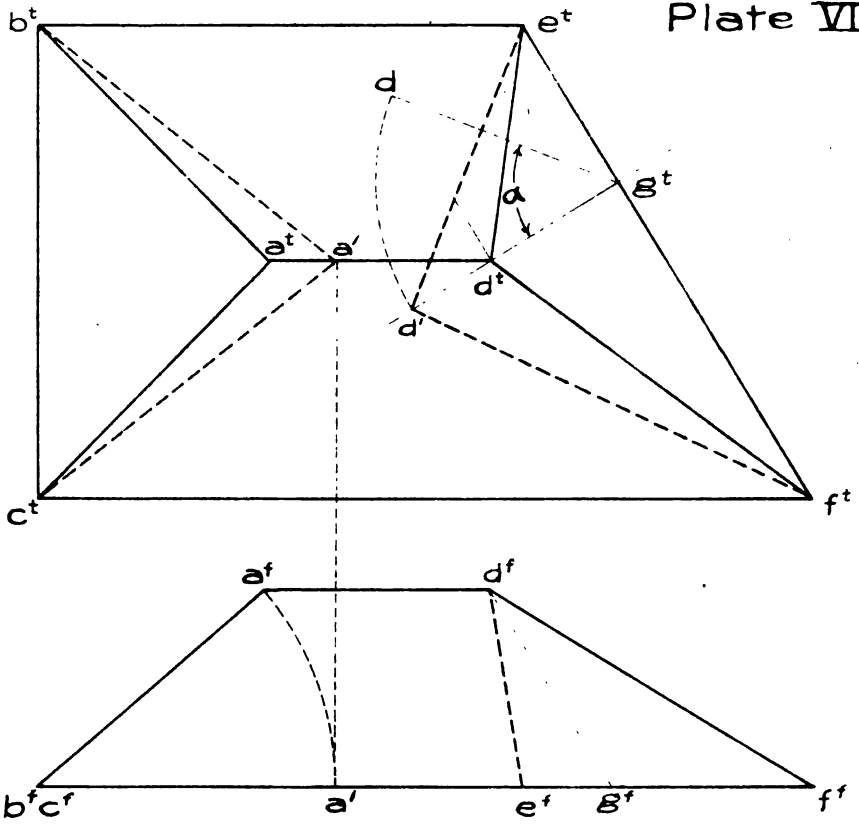


Fig. 15

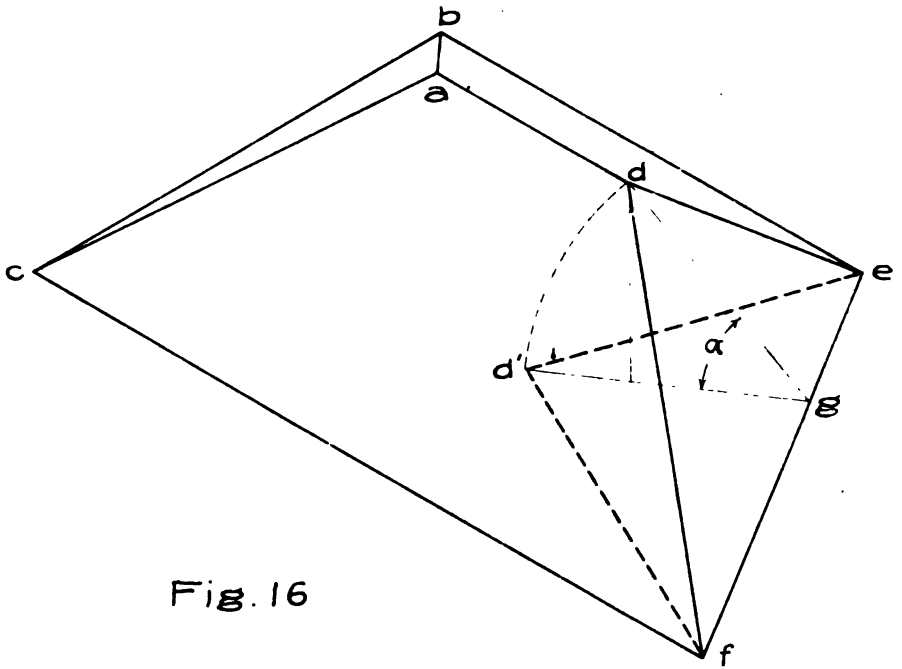


Fig. 16

Plate VII.

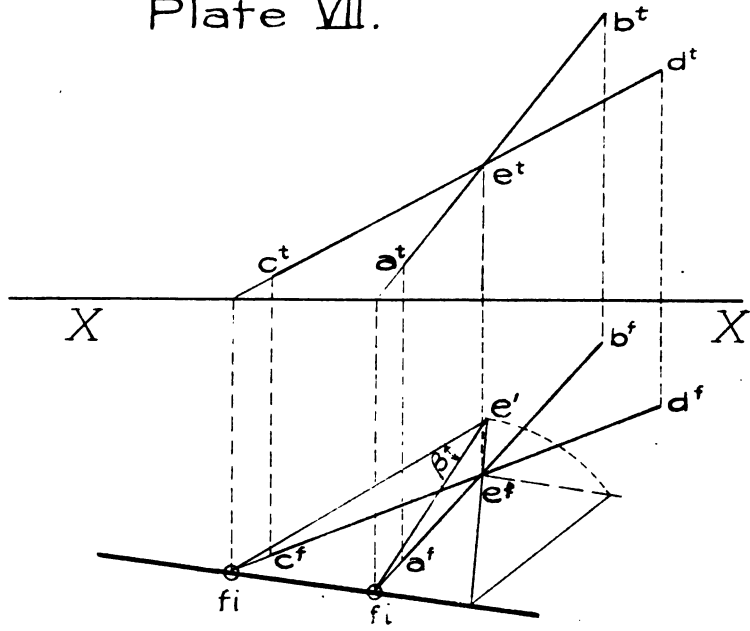


Fig. 17

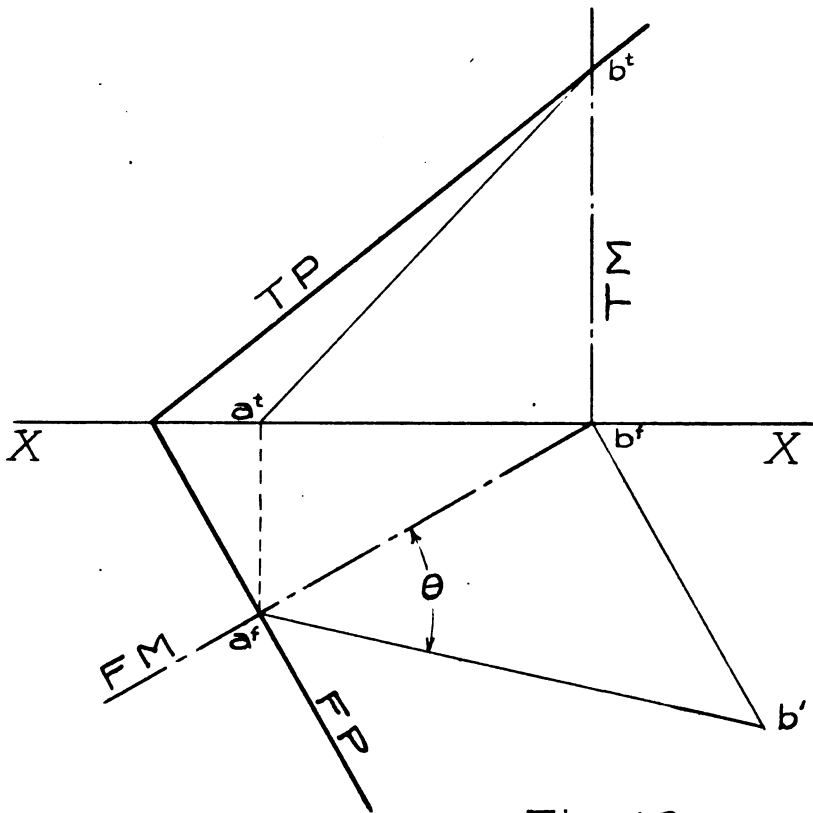


Fig. 18



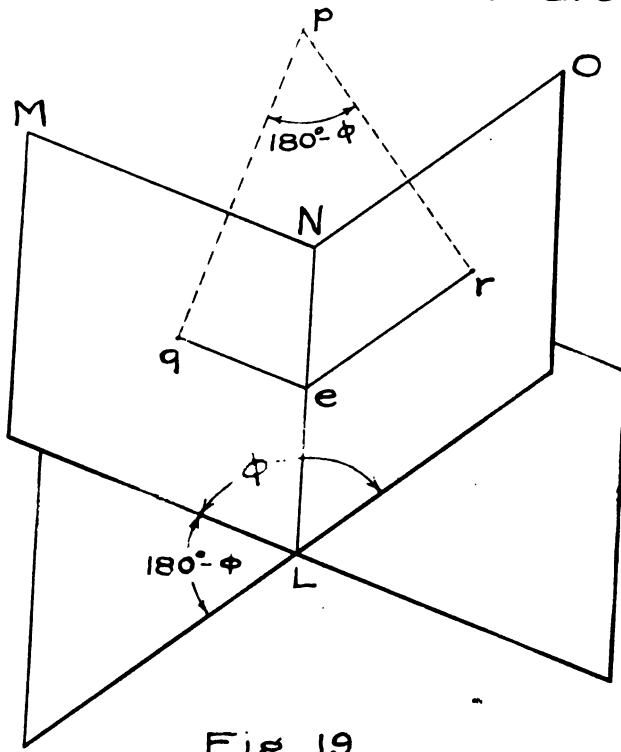


Fig. 19

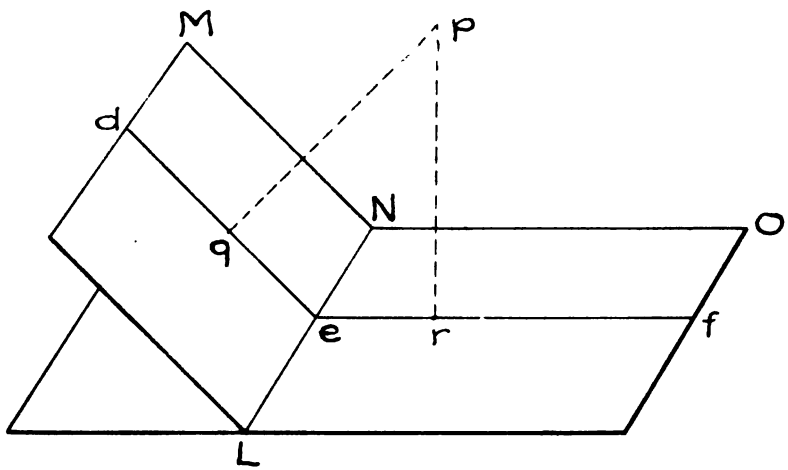


Fig. 20

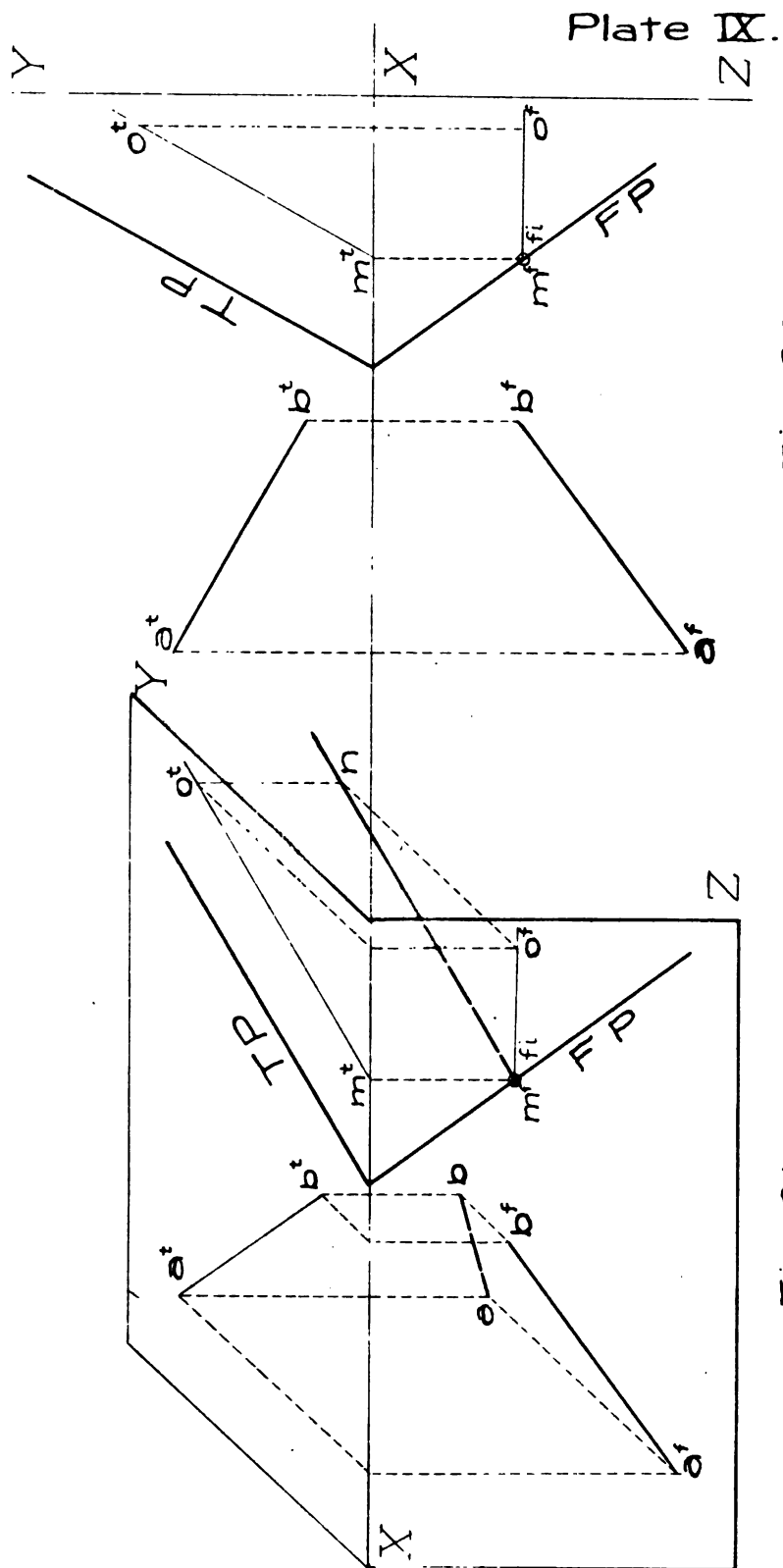


Fig. 22

Fig. 21

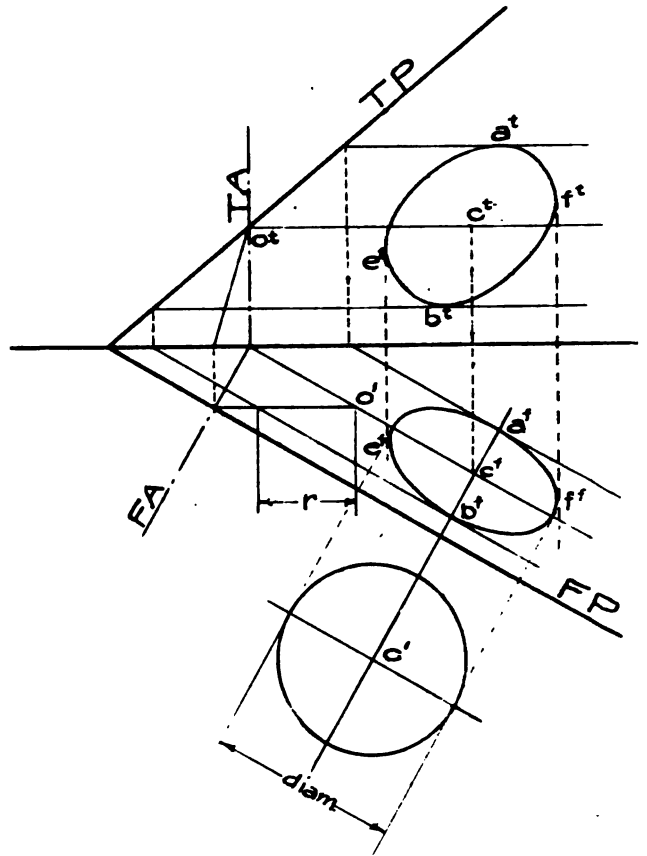


Fig. 25

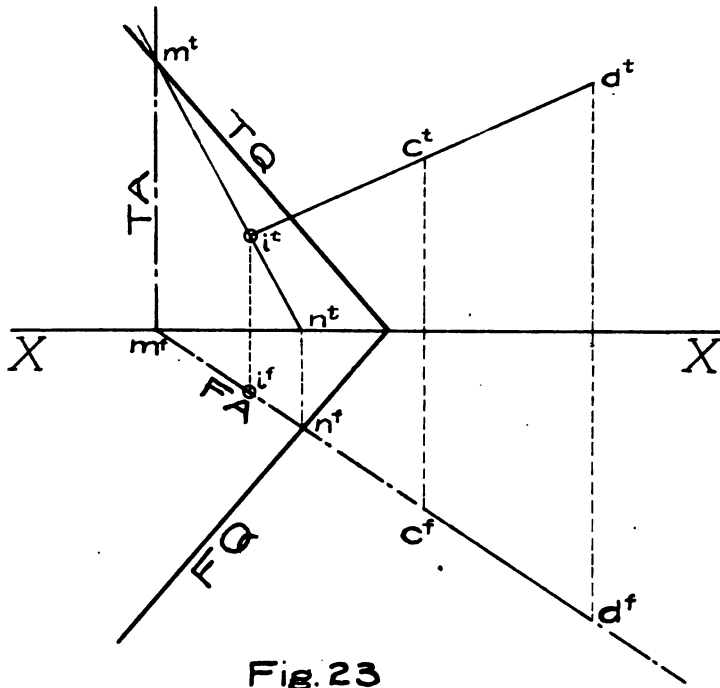


Fig. 23

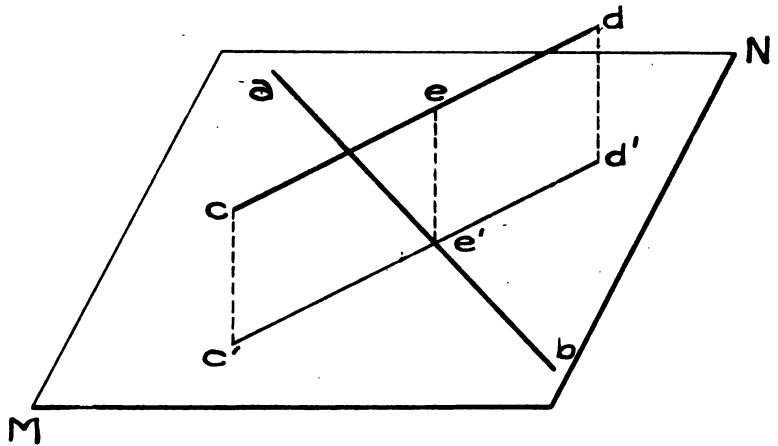
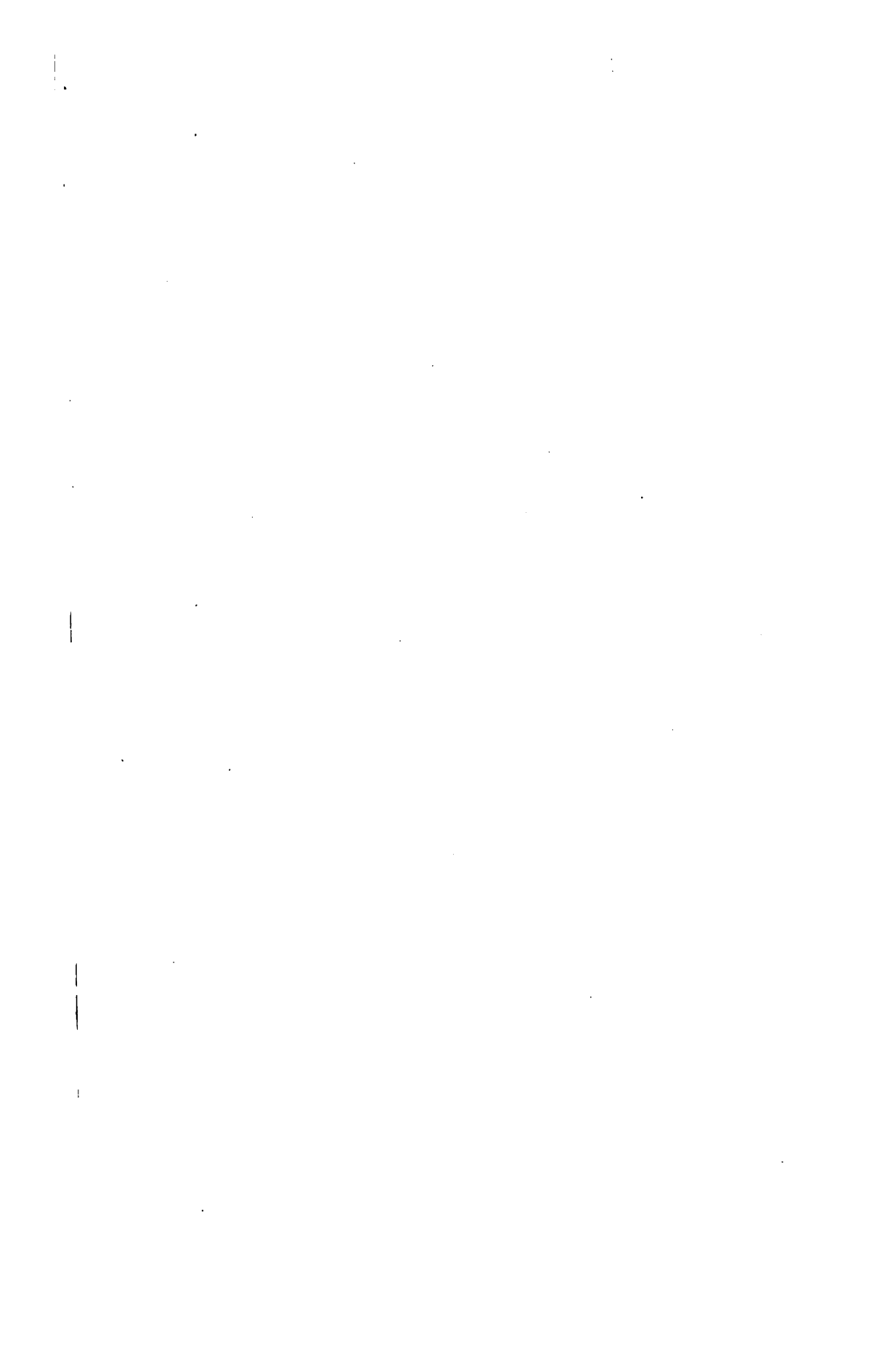


Fig. 24



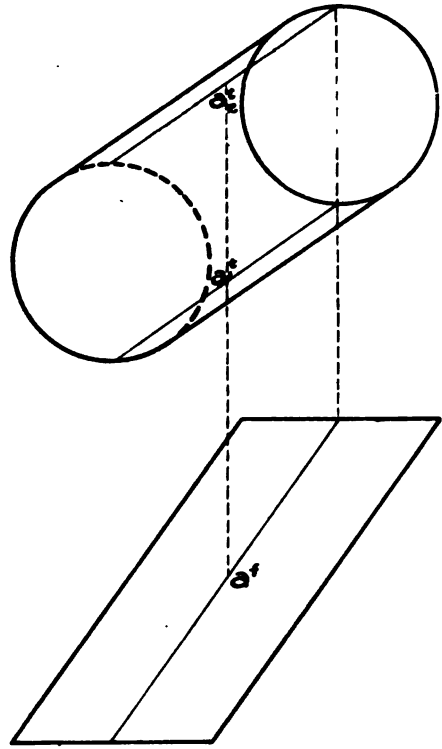
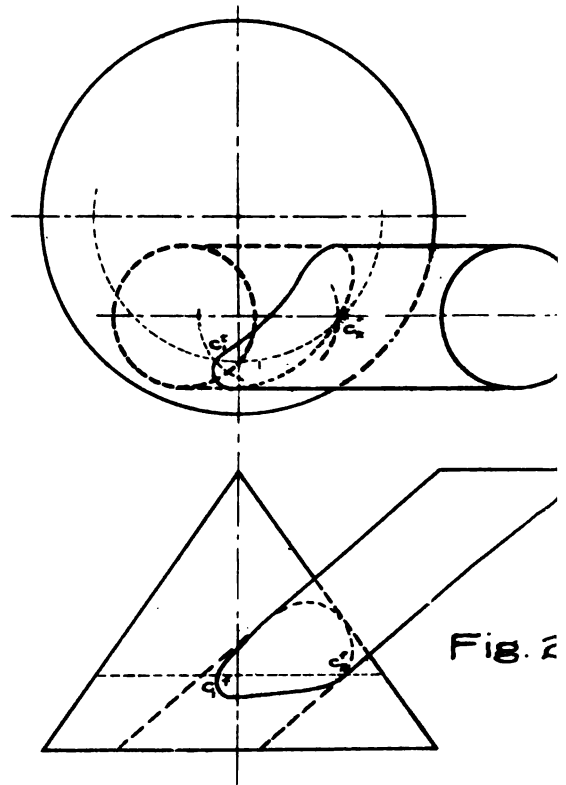


Fig. 26



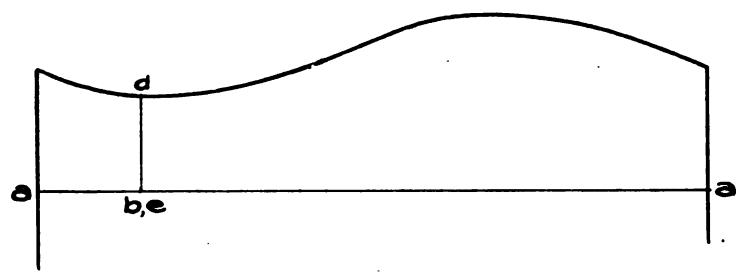
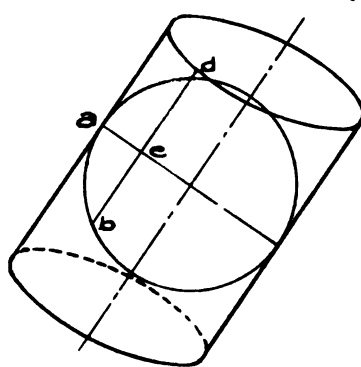


Fig. 27

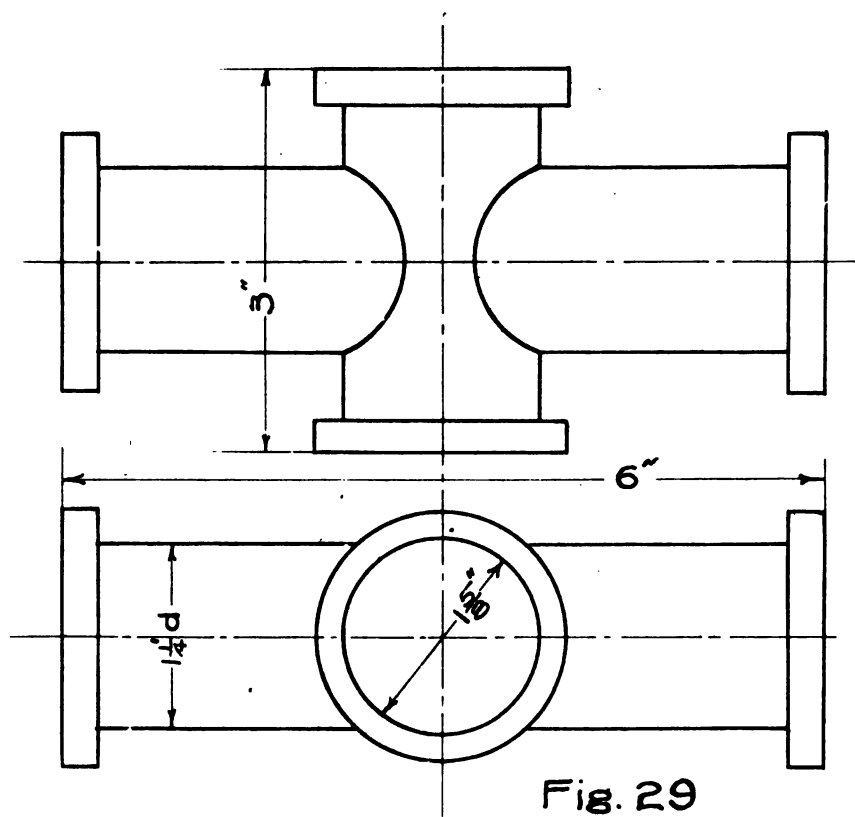


Fig. 29

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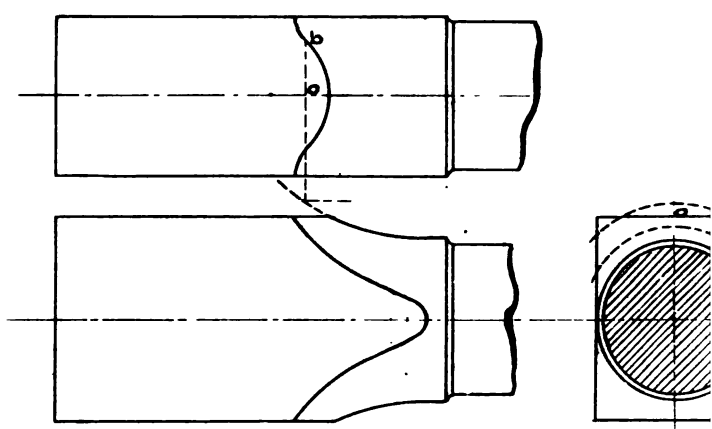


Fig. 30

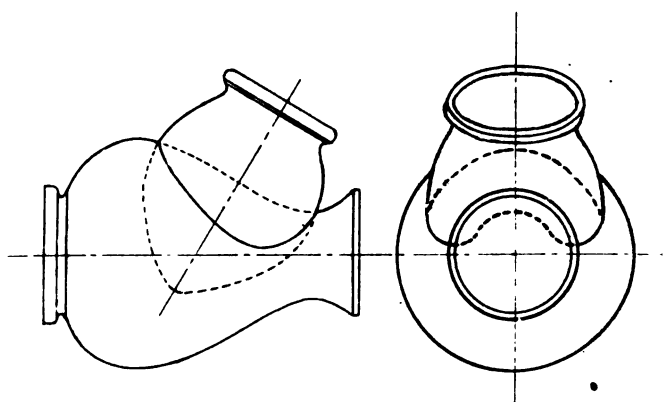


Fig. 32

Plate XII.

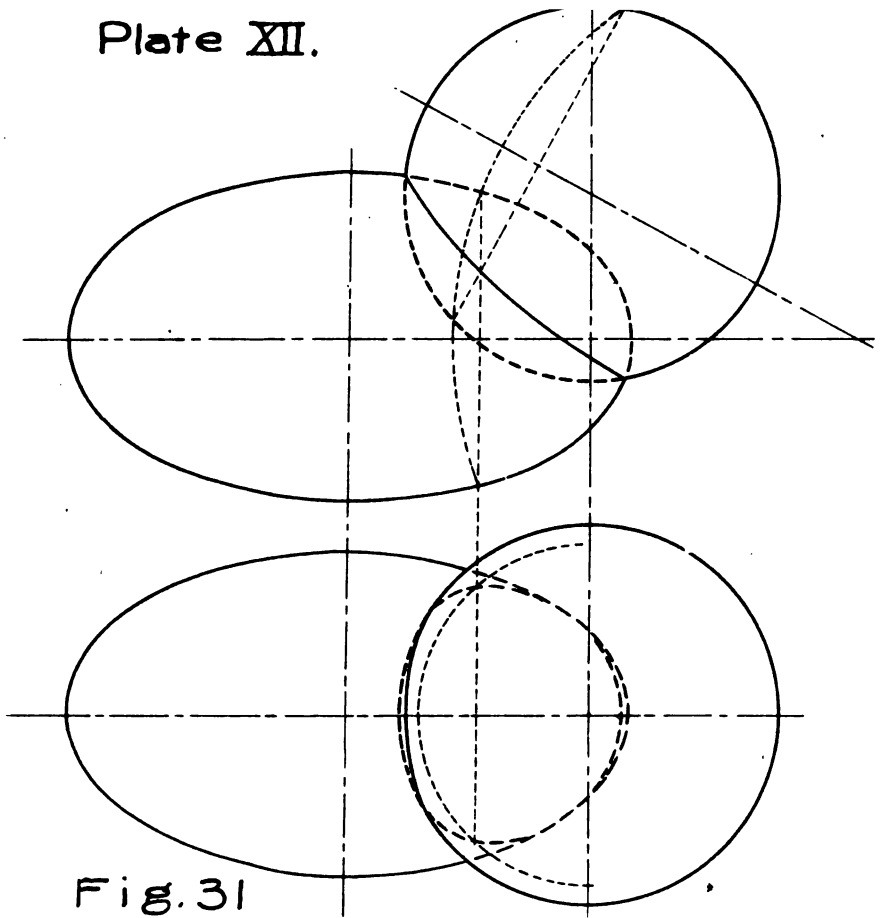


Fig. 31

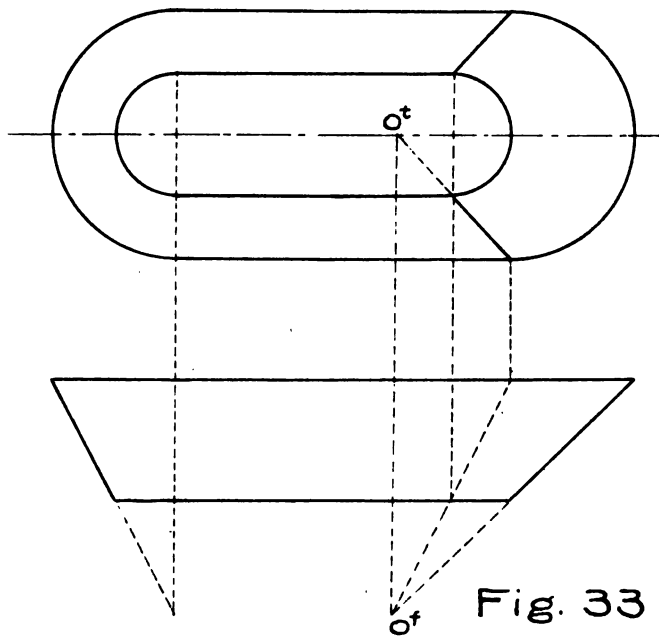


Fig. 33



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